Household Debt, Growth and Inequality

Gaël Giraud

AFD,French Development Agency, CNRS & Energy and Prosperity Chair
with Matheus GRASSELLI (Mc Master University, Fields Institute, Toronto)

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- First: growth, second: distribution.
  "*Changes in debt are 'pure redistributions' which should have no significant macro-economic effects*" (Bernanke, 2000, p. 24).

- A perfectly flexible market determines a unique static equilibrium (Debreu (1970)).

- This static equilibrium is always first-best efficient (Pareto).

- No ordinal axiomatization of any concept of justice (the cardinality curse).

- No room for justice?

- However, a perfectly flexible market is efficient only if there are no
  - externalities, increasing returns to scale, incomplete markets
- Incomplete markets are (almost) always second-best inefficient (Geanakoplos-Polemarchakis (1986))
- Incomplete markets exhibit a huge indeterminacy of equilibria (Mas-Colell (1984))
- Financial innovation does not necessarily improve the (third-best) efficiency of incomplete markets (Elul (1995)).

Moreover...

- With incomplete markets static equilibria may fail to exist in a robust manner... (Momi (2000)).
- There exist only 3 types of equilibria (Giraud - Pottier (2016)):
  - with inflation and growth
  - deflation without growth (Irving Fisher)
  - with speculative bubbles on financial markets
- A crisis like 2008 cannot occur at a static equilibrium (of only as a "black swan" (Taleb (2009), Giaud-Pottier (2009)).
- How do we know that South-African economy is already at equilibrium (if any)?

Moreover...(2)

- Mertens and Dhillon (1999) and Fleurbaey and Maniquet (2006) provide ordinal axiomatizations of (relative) Utilitarianism and the Maximin.
- A Utilitarian solution need not coincide with the “market solution”.
- Refinements of the mere Pareto-optimality notion (e.g., nucleolus, Shapley value, Harsanyi value...) need not coincide with Arrow-Debreu equilibria.
- Justice makes sense and does not emerge spontaneously from market interactions. (SDG 10.)

- Banchard (PIIE, 2016):
  I see the current DSGE models as seriously flawed...
- Romer (2016):
  For more than three decades, macroeconomics has gone backwards...
- Kocherlakota (2016):
  ...we simply do not have a settled successful theory of the macroeconomy. The choices made 25-40 years ago - made then for a number of excellent reasons - should not be treated as written in stone or even in pen.

Need for change in our analytical framework.

Articulation between ecological sustainability / inequality / prosperity.

Main takeaways

1) Need to incorporate the dynamics of private debts

2) An increase of income inequality is a signal for a decline in growth in the long-run.

3) $r > g$ is a necessary condition for the stability of a debt-deflationary long-run equilibrium with exploding inequalities.
An increase in K/Y reinforces its stability.

I. Critics of Piketty (2014)

- $Y_n = (Y_n - W) + W$ (total income equals capital income plus labor income)
- $r_k = \frac{(Y_n - W)}{pK}$ (rate of return on capital)
- $\alpha_k = \frac{Y_n - W}{Y_n}$ (capital share of total income)
- $\beta_k = \frac{pK}{Y_n}$ (capital-to-income ratio)

I. Critics of Piketty (2014)

- First "fundamental law of capitalism" $\alpha_k = r_k \beta_k$: trivial accounting equation.
- Second "law'" is false : $\beta \to s/g$ (Stiglitz, Acemoglu, Varoufakis, Taylor, Giraud...)
- $r_k > g$ well-known, and so what? (Acemoglu, Mankiw, IMF...). Confusion between r and r_k.
- Cambridge controversy about capital (Varoufakis, Giraud, Taylor...)
- A model without money? Is money neutral? No endogenous creation of credit by banks?

II. Debts and credit

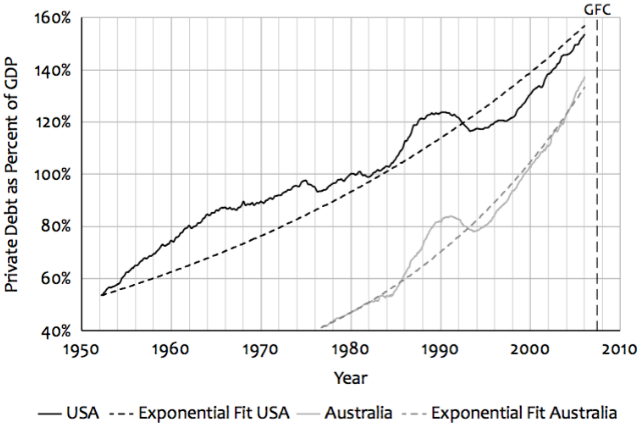

Figure 6. The exponential increase in debt to GDP ratios till 2006

Figure: Keen (2017)

Debts and credit

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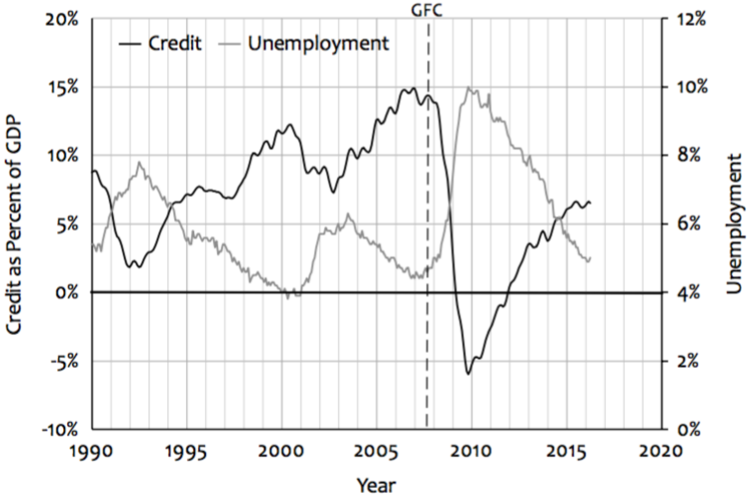


Figure: Keen (2017)

Debts and credit

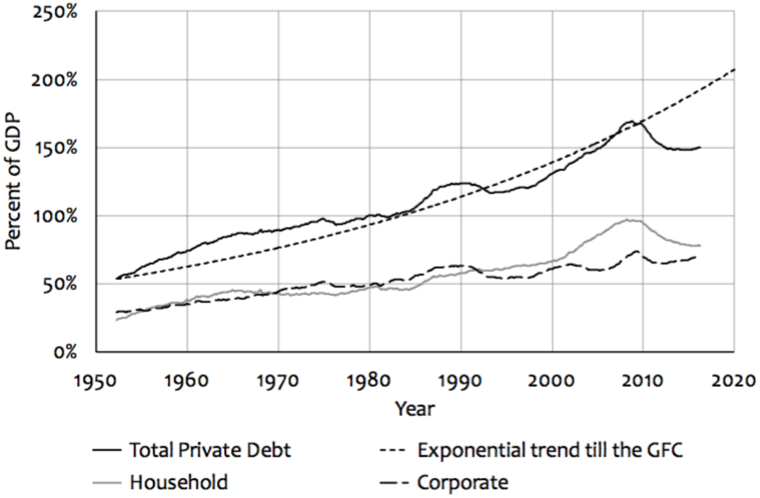


Figure: Households vs firms. Keen (2017)

Debts and credit

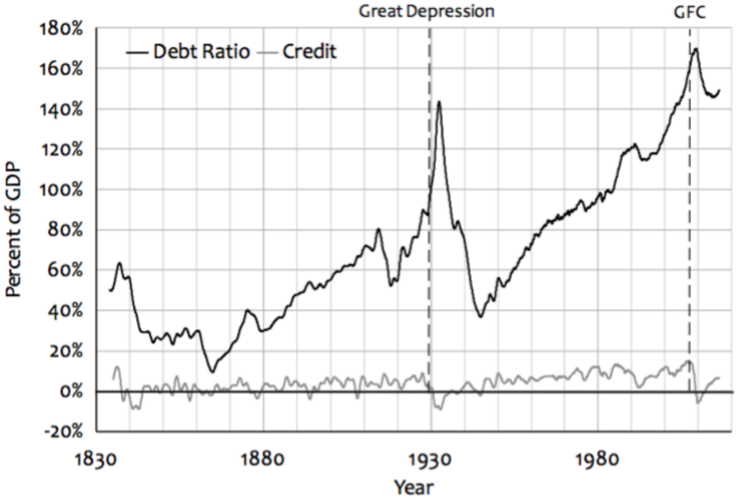


Figure: Keen (2017)

Debts and credit

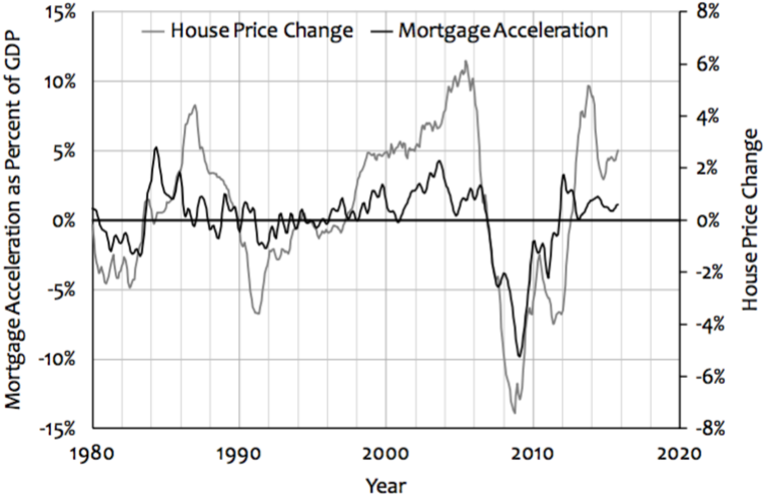


Figure: Keen (2017)

Debts and credit

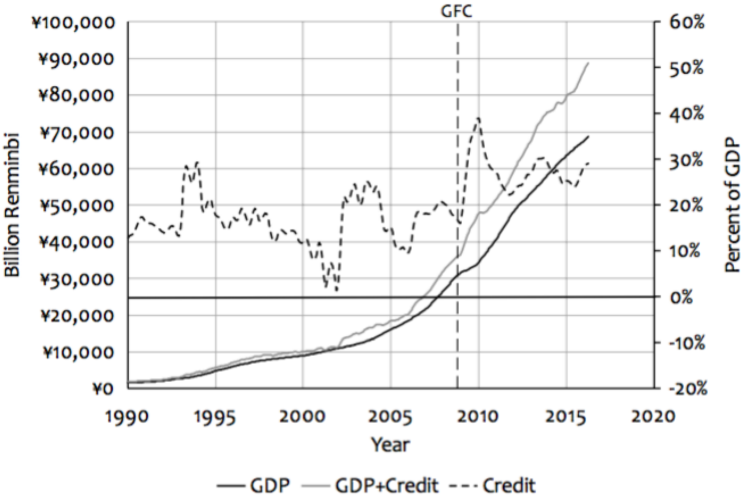


Figure: China (Keen (2017))

Debts and credit

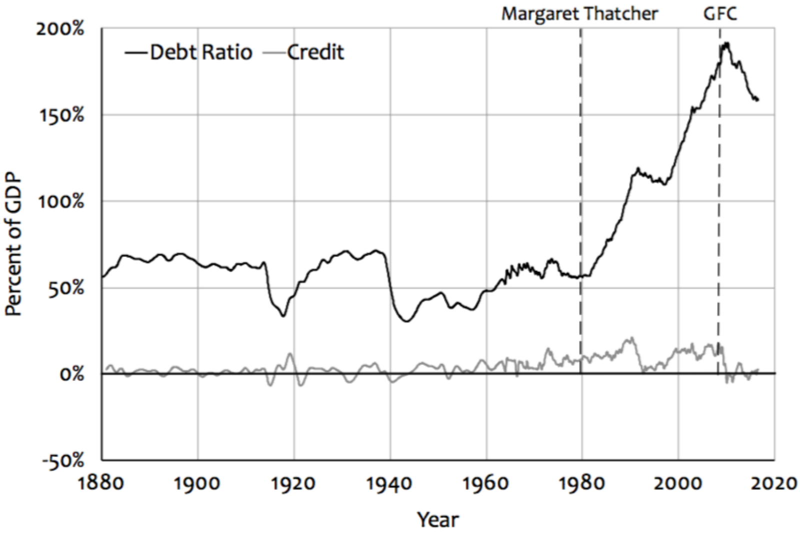


Figure: UK (Keen (2017))

III. An alternative approach

Suppose our economy is a ball...

McIsaac et al. (2016).

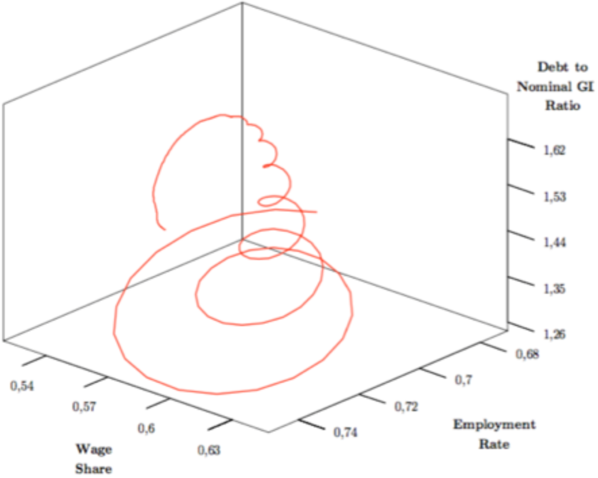


Figure: Les trajectoires du scénario *Business-As-Usual*.

McIsaac et al. (2016).

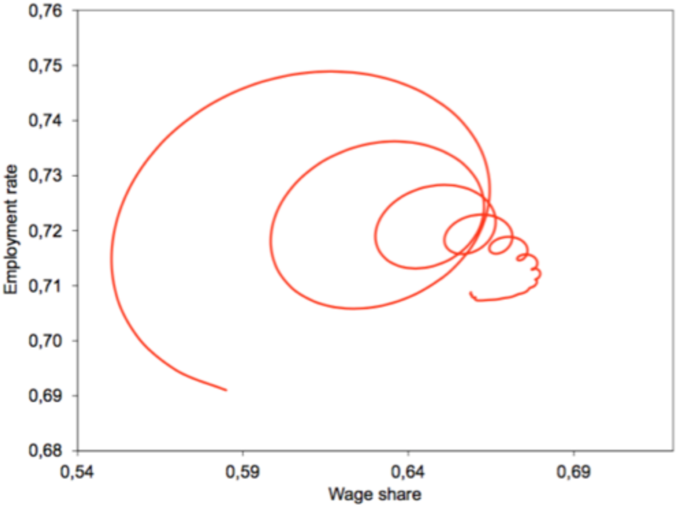


Figure: Trajectoires du scénario Burke et al. (2015).

McIsaac et al. (2016).

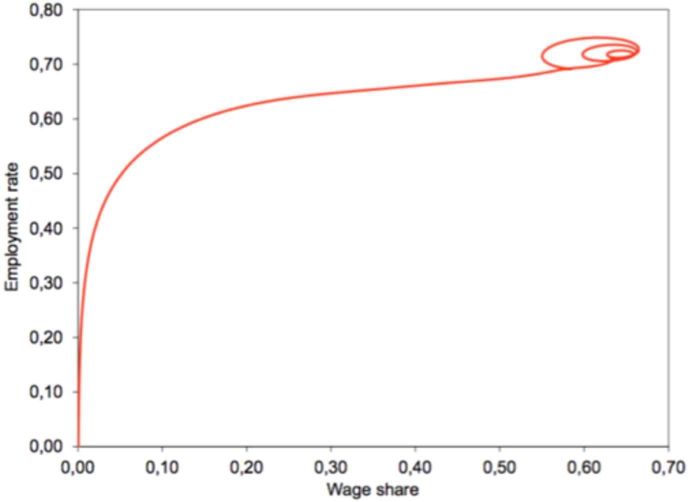
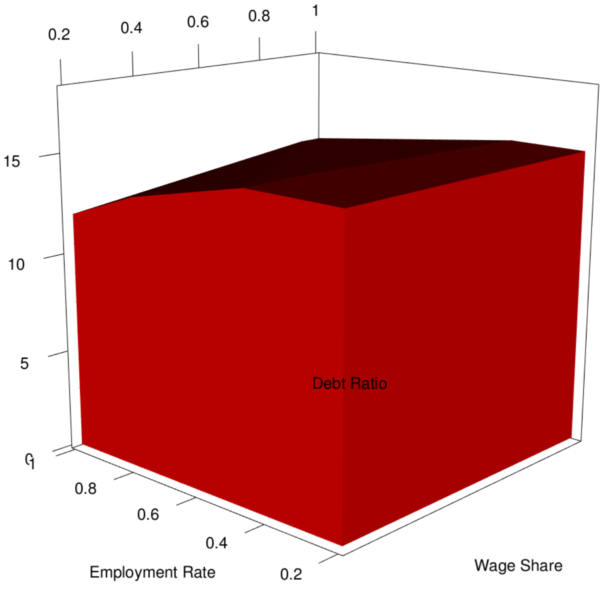
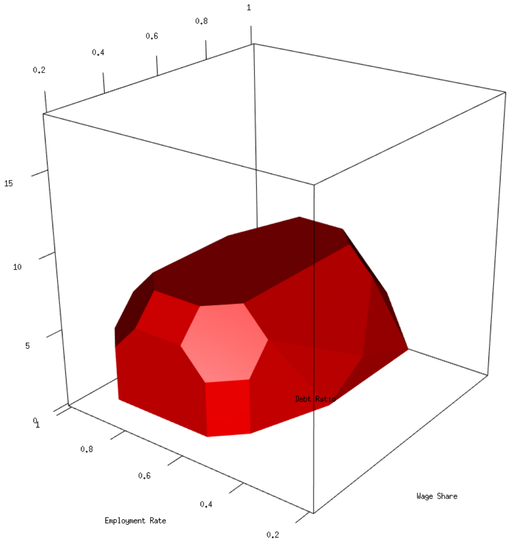


Figure: Portrait de phase de la combinaison de Burke *et al.* et de Dietz-Stern case.

The basin of attraction of a "good" equilibrium without climate change (McIsaac et al. (2016)).



With climate change (McIsaac et al. (2016)).



Properties

- Stock-Flow consistency (Godley-Lavoie (2012)).
- Money is non-neutral and endogenous (Diamond-Dybvig, Tobin, Bank of England...)
- Collapses are possible
- Long-run dynamics out-of-equilibrium.
- Multiple equilibria.
- Key role of private debts.

Giraud-Grasselli (2017)

- In general, there are 3 types of long-run equilibria.
- One equilibrium is not locally stable.
- One stable equilibrium "à la Solow".
  $g = \alpha + \beta$ + "Golden rule"
  $\lambda \to$ NAIRU (Tobin).
  Inequality remains stable
- One stable equilibrium leads to a collapse
  Inequality explodes.
  $\lambda \to 0$.

SFC table for the dual Akerlof-Stiglitz (1969) model

	Households	Firms		Banks	Sum
Balance sheet					
Capital stock		$+pK$			pK
Deposits	$+M_h$	$+M_f$		$-(M_h + M_f)$	0
Loans	$-L_h$	$-L_f$		$+(L_h + L_f)$	0
Sum (Net worth)	X_h	X_f		X_b	X
Transactions		Current	Capital		
Consumption	$-pC_h$	$+pC$		$-pC_b$	0
Investment		$+pI$	$-pI$		0
Accounting memo [GDP]		$[pY]$			
Depreciation		$-p\delta K$	$+p\delta K$		0
Wages	$+\mathrm{w}\ell$	$-\mathrm{w}\ell$			0
Interest on loans	$-rL_h$	$-rL_f$		$+r(L_h + L_f)$	0
Interest on deposits	$+rM_h$	$+rM_f$		$-r(M_h + M_f)$	0
Dividends	$+\Delta_b$			$-\Delta_b$	0
Financial balances	S_h	S_f	$-pI + p\delta K$	S_b	0
Flows of funds					
Change in capital stock		$+p(I - \delta K)$			$+p(I - \delta K)$
Change in deposits	$+\dot{M}_h$	$+\dot{M}_f$		$-(\dot{M}_h + \dot{M}_f)$	0
Change in loans	$-\dot{L}_h$	$-\dot{L}_f$		$+(\dot{L}_h + \dot{L}_f)$	0
Column sum	S_h	S_f		S_b	$+p(I - \delta K)$
Change in net worth	$\dot{X}_h = S_h$	$\dot{X}_f = S_f + \dot{p}K$		$\dot{X}_b = S_b$	$\dot{X} = \dot{p}K + p\dot{K}$

Table: SFC table for the dual Akerlof-Stiglitz (1969) model.

Dual Akerlof-Stiglitz (1969) model - Definitions

- $D_h := L_h - M_h$ and $D_f := L_f - M_f$
  Assume $\Delta_b = r(D_h + D_f)$ and $C_b = 0$.
- $\Rightarrow S_b = 0$, so we take $X_b = 0$, $\Rightarrow D_h = -D_f$.
- $$\begin{aligned} \dot{D}_h &= pC_h - \mathrm{w}\ell + rD_h - r(D_h + D_f) \\ &= pY - pI - \mathrm{w}\ell - rD_f = -\dot{D}_f. \end{aligned}$$
- "Deposits create loans"...

Dual Akerlof-Stiglitz (1969) model - Definitions

- $\omega := W/(pY)$, $d_h := D_h/(pY)$
- Assume consumption $C := c(\omega - rd)Y$
  Disposable income $(\omega - rd)$.
- $I := Y - C$,

$$\dot{K} = Y - C - \delta K = \left(\frac{1 - c(\omega - rd)}{\nu} - \delta \right) K$$

where $\nu := K/Y$ is a constant capital-to-output ratio.

Differential Equations

- Assume further a wage-price dynamics (short-run Phillips curve, Gordon (2012), Mankiw (2010), ECB...)

$$\frac{\dot{w}}{w} = \Phi(\lambda) + \gamma \left(\frac{\dot{p}}{p} \right)$$

$$i(\omega) = \frac{\dot{p}}{p} = \eta_p(m\omega - 1),$$

  for a constant mark-up factor $m \geq 1$.
  Imperfect competition on commodity market.

Dual Akerlof-Stiglitz (1969) model - Differential Equations

- The model can now be described by the following system

$$
\begin{aligned}
\dot{\omega} &= \omega[\Phi(\lambda) - \alpha - (1-\gamma)i(\omega)] \\
\dot{\lambda} &= \lambda\left[\frac{1-c(\omega-rd_h)}{\nu} - (\alpha+\beta+\delta)\right] \\
\dot{d}_h &= d_h\left[r - \frac{1-c(\omega-rd_h)}{\nu} + \delta - i(\omega)\right] + c\left(\omega - rd_h\right) - \omega.
\end{aligned}
$$

Dual Akerlof-Stiglitz (1969) model - Equilibria

- Analogously to the original Akerlof-Stiglitz (1969)/Goodwin (1967)/Van der Ploeg (1974) models, there is a good equilibrium characterized by

$$\overline{\omega}_1 = \eta + r\Big[\frac{1-\eta-\nu(\alpha+\beta+\delta)}{\alpha+\beta+i(\overline{\omega}^1)}\Big].$$
$$\overline{\lambda}_1 = \Phi^{-1}\big(\alpha+(1-\gamma)i(\overline{\omega}^1)\big).$$
$$\overline{d}_1 = \frac{1-\eta-\nu(\alpha+\beta+\delta)}{\alpha+\beta+i(\overline{\omega}^1)},$$

where $\eta_1 := c^{-1}\big(1-\nu(\alpha+\beta+\delta)\big)$.

- It also exhibits a bad equilibrium of the form $(0, 0, +\infty)$.
- Both equilibria can be locally stable for some parameter values, but *not* at the same time.
- There's also an equilibrium of the form $(\overline{\omega}_3, 0, \overline{d_{h3}})$.

Example 1: convergence to the interior (good) equilibrium (phase space)

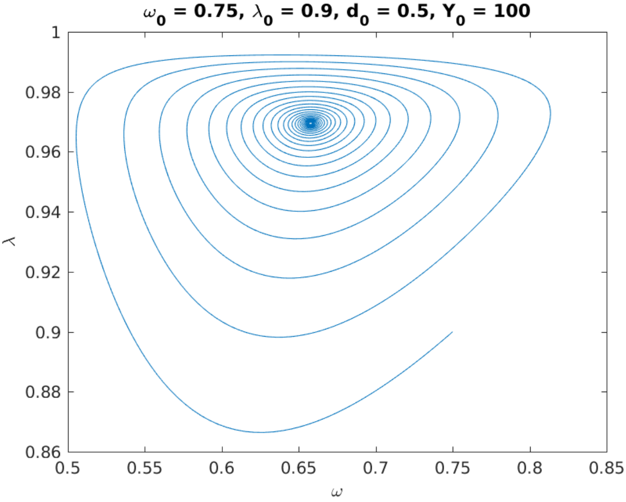
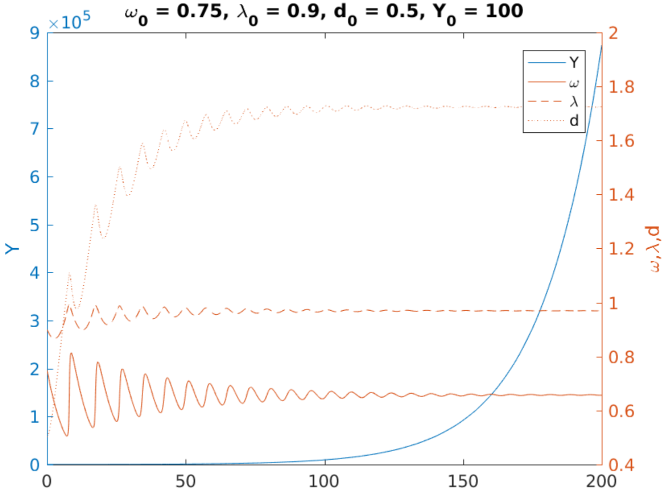


Figure: $\nu = 3$, $\eta_p = 0.35$, $\gamma = 0.8$

Example 1: convergence to the interior equilibrium (time)



Example 2: business cycles (phase space)

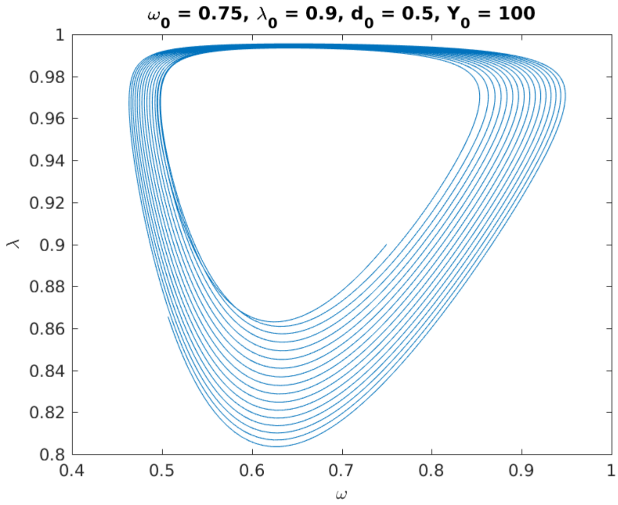
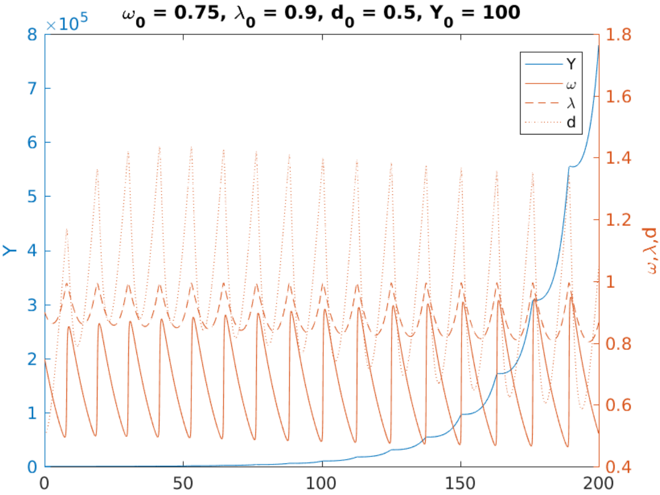


Figure: $\nu = 3$, $\eta_p = 0.45$, $\gamma = 0.96$

Example 2: business cycles (time)



Example 3: convergence to debt-deflationary equilibrium (phase)

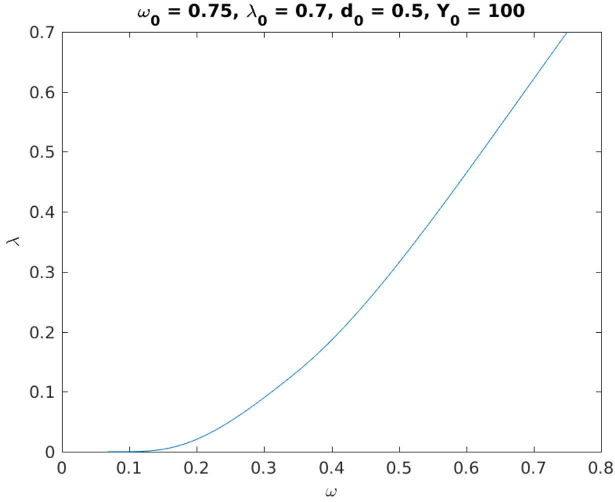
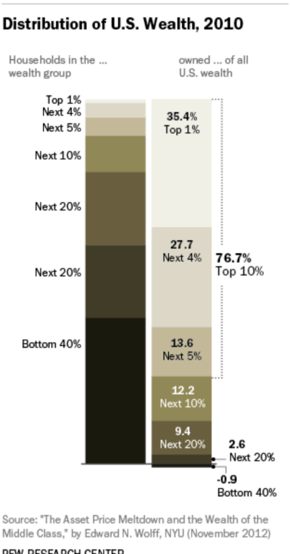


Figure: $\nu = 15$, $\eta_p = 0.35$, $\gamma = 0.8$

Workers versus investors - motivation



Workers versus investors - modelling

- Two different classes of households, namely workers and investors, with wealth given by

$$X_w = -D_w$$
$$X_i = E_f + E_b - D_i.$$

- Budget constraint that

$$\dot{D}_w = pC_w - \mathrm{w}\ell + rD_w$$
$$\dot{D}_i = pC_i - r_k pK - \Delta_b + rD_i.$$

- Finally, assume that consumption is of the form $C_w = c_w(y_w)Y$ and $C_i = c_i(y_i)Y$ with

$$\frac{\partial c_w}{\partial y_w}(\omega - rd_w) > \frac{\partial c_i}{\partial y_i}(r_k\nu - rd_i).$$

SFC table for the two-class Akerlof-Stiglitz (1969) model

	Workers	Investors	Firms		Banks	Sum
Balance sheet						
Capital stock			$+pK$			pK
Deposits	$+M_w$	$+M_i$	$+M_f$		$-(M_w + M_i + M_f)$	0
Loans	$-L_w$	$-L_i$	$-L_f$		$+(L_w + L_i + L_f)$	0
Equities		$+p^e E$	$-p^e E$			0
Sum (Net worth)	X_w	X_i	X_f		X_b	X
Transactions			Current	Capital		
Consumption	$-pC_w$	$-pC_i$	$+pC$		$-pC_b$	0
Investment			$+pI$	$-pI$		0
Accounting memo [*GDP*]			$[pY]$			
Wages	$+\mathrm{w}\ell$		$-\mathrm{w}\ell$			0
Depreciation			$-p\delta K$	$+p\delta K$		0
Interest on loans	$-rL_w$	$-rL_i$	$-rL_f$		$+r(L_w + L_i + L_f)$	0
Interest on deposits	$+rM_w$	$+rM_i$	$+rM_f$		$-r(M_w + M_i + M_f)$	0
Dividends		$+r_k pK + \Delta_b$	$-r_k pK$		$-\Delta_b$	0
Financial balances	S_w	S_i	S_f	$-pI + p\delta K$	S_b	0
Flows of funds						
Change in capital stock			$+p(I - \delta K)$			$p(I - \delta K)$
Change in deposits	$+\dot{M}_w$	$+\dot{M}_i$	$+\dot{M}_f$		$-(\dot{M}_w + \dot{M}_i + \dot{M}_f)$	0
Change in loans	$-\dot{L}_w$	$-\dot{L}_i$	$-\dot{L}_i$		$+(\dot{L}_w + \dot{L}_i + \dot{L}_f)$	0
Change in equities		$+p^e \dot{E}$	$-p^e \dot{E}$			0
Column sum	S_w	S_i	S_f		S_b	$p(I - \delta K)$
Change in net worth	$\dot{X}_w = S_w$	$\dot{X}_i = S_i + \dot{p}^e E$	$\dot{X}_f = S_f - \dot{p}^e E + \dot{p}K$		$\dot{X}_b = S_b$	$\dot{X} = \dot{p}K + p\dot{K}$

Table: SFC table for the workers and investors model.

Return on capital and external financing

- Assume firms retain profits according to a constant retention rate Θ, leading to an endogenous return on capital given by

$$\begin{aligned} r_k := r_k(\omega, d_w, d_i) &= \frac{\Theta(pY - \mathrm{w}\ell - rD_f - p\delta K)}{pK} \\ &= \frac{\Theta}{\nu}\left(1 - \omega + r(d_w + d_i) - \delta\nu\right), \end{aligned}$$

- Savings by firms are endogenous

$$S_f = (1-\Theta)(pY - \mathrm{w}\ell - rD_f - p\delta K) = pY - \mathrm{w}\ell - rD_f - p\delta K - r_k pK$$

- Therefore, the amount to be raised externally by firms is

$$\begin{aligned} p(I - \delta K) - S_f &= pI - pY + \mathrm{w}\ell + rD_f + r_k pK \\ &= \big(\omega - r(d_i + d_w) - c + r_k\nu\big)pY, \end{aligned}$$

- As in the Akerlof-Stiglitz (1969) model, this is raised solely through new loans from the banking sector.

The main dynamical system

- Aggregate consumption

$$c(\cdot) \equiv c(\omega, d_w, d_i) = c_w(\omega - rd_w) + c_i(r_k\nu - rd_i),$$

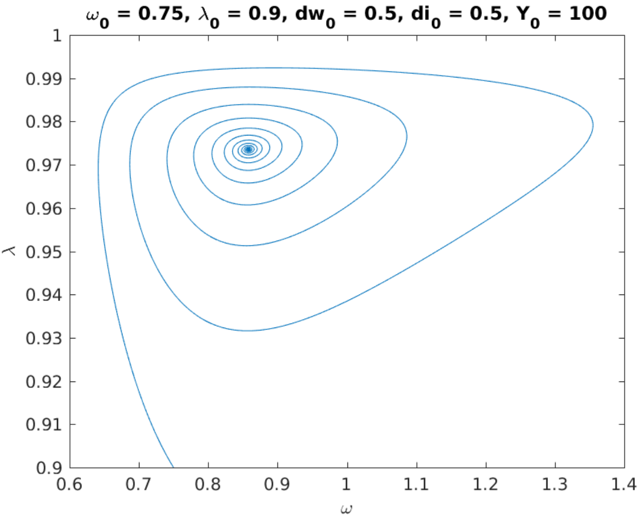
- Dynamical system

$$\begin{aligned}
\dot{\omega} &= \omega[\Phi(\lambda) - \alpha - (1-\gamma)i(\omega)] \\
\dot{\lambda} &= \lambda\left[\frac{1-c(\cdot)}{\nu} - (\alpha+\beta+\delta)\right] \\
\dot{d}_w &= d_w\left[r + \delta - \frac{1-c(\cdot)}{\nu} - i(\omega)\right] + c_w(\omega - rd_w) - \omega \\
\dot{d}_i &= d_i\left[r + \delta - \frac{1-c(\cdot)}{\nu} - i(\omega)\right] + c_i(r_k\nu - rd_i) - r_k\nu
\end{aligned}$$

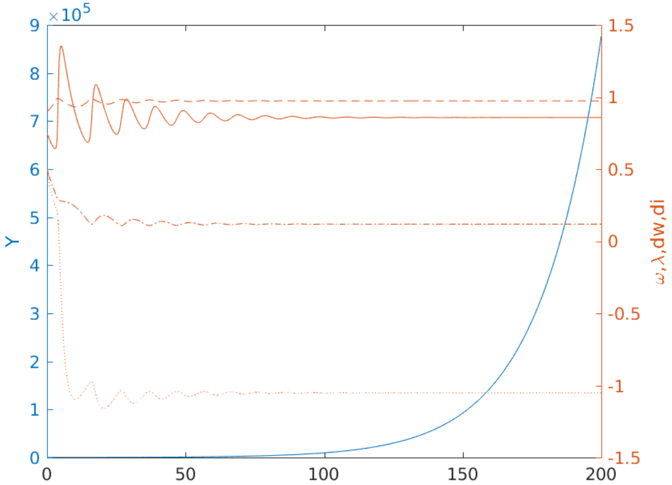
Equilibria

- With considerable more work, it is possible to show that the system exhibits a class of good equilibria of the form $(\overline{\omega}_1, \overline{\lambda}_1, \overline{d}_{w1}, \overline{d}_{i1})$ typically (but not always) satisfying $\overline{d}_{w1} > 0$ and $\overline{d}_{i1} < 0$.
- In addition, the system admits a class of bad equilibria $(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_{w2}, \overline{d}_{i2}) = (0, 0, \pm\infty, \pm\infty)$
  Which are locally asymptotically stable only if $r_k > g$.
- Finally, it also exhibits deflationary equilibria of the form $(\overline{\omega}_3, 0, \overline{d}_{w3}, \overline{d}_{i3})$, where $\overline{d}_{w3}$ and $\overline{d}_{i3}$ can be either finite of infinite.

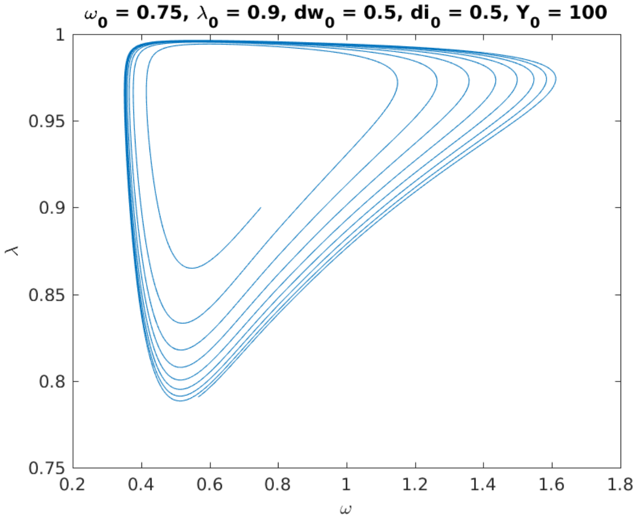
Example 4: convergence to the interior equilibrium (phase space)



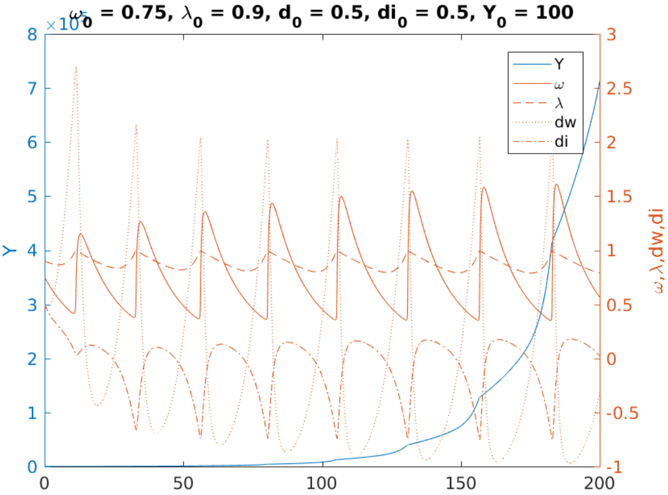
Example 4: convergence to the interior equilibrium (time)



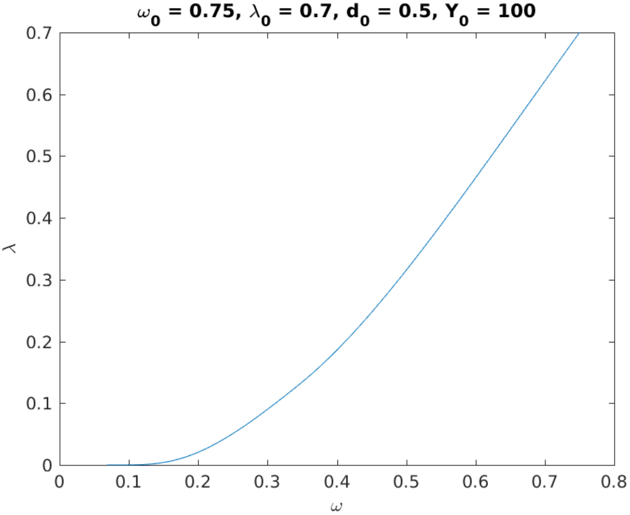
Example 5: business cycles (phase space)



Example 5: business cycles (time)



Example 6: convergence to debt-deflationary equilibrium (phase)



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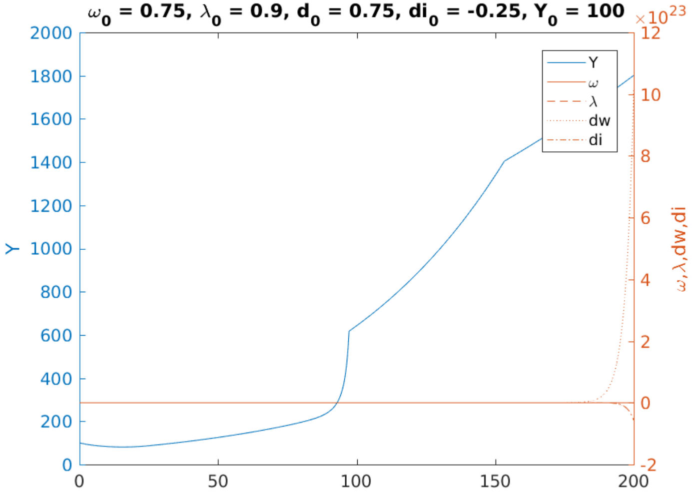

Long-run inequality

- Income shares of nominal output for workers, investors, and firms:

$$y_w = \frac{Y_w^n}{pY} = \omega - rd_w$$
$$y_i = \frac{Y_i^n}{pY} = r_k\nu - rd_i$$
$$\pi_r = \frac{\Pi_r}{pY} = (1-\Theta)(1-\omega-rd_f-\delta\nu),$$

$\Rightarrow$ income share of capital

$$y_c = y_i + \pi_r = 1 - \omega + rd_\omega - \delta\nu = 1 - y_\omega - \delta\nu.$$

- Easy to see: the growth rate of real income for all three sectors coincide at the interior equilibrium = $\alpha + \beta$.

Inequality as a hallmark of inefficiency

- However, at each of the equilibria $(\overline{\omega}_2, \overline{\lambda}_2, \overline{d}_{w2}, \overline{d}_{i2}) = (0, 0, \pm\infty, \pm\infty)$ we observe divergence in income between workers and capitalists.
- For example, if $d_w \to +\infty$ and $d_i \to -\infty$, then $y_w \to -\infty$, $y_i \to +\infty$, $\pi_r \to -\infty$, whereas $y_c \to +\infty$.
- Similarly, whenever $d_w \to +\infty$, we have $x_w \to -\infty$ and $x_i \to +\infty$.
- At the deflationary equilibrium $(\overline{\omega}_3, 0, \overline{d}_{w3}, \overline{d}_{i3})$, the income shares are $r_k\nu - r\overline{d}_{i3}$ and $\overline{\omega}^3 - r\overline{d}_{w3}$.
- An artifact of the fact that prices are falling faster than real output $Y \to \overline{\lambda}_3 N/a = 0$.
- Real income of both populations collapse, so *both* types of households end up ruined!

Concluding remarks

- We provided a stock-flow consistent model for debt dynamics of workers and investors.
- When the economy converges to an equilibrium with finite debt ratios, the income ratio between the two classes is constant.
- Increasing income (and wealth) inequality is a signature of convergence to the bad equilibrium with infinite debt ratios.
- In future work we explore the effects of default, variable capacity utilization, substitutability between capital and labor, and of migration between classes à la Acemoglu (2014).
- THANK YOU!