# Values, Volumes, and Price-Volume decompositions: Some Issues Raised (Again) by the Health Crisis

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*Online appendix*

## Equivalent incomes with changing preferences

We use the same model as in the text of the article to compare different implementations of the equivalent income method with a combination of productivity changes and preference changes. As in the text of the article, we have two sectors that use only labor input in quantities $l_1$ and $l_2$ summing to 1, with productivities $\pi_1$ and $\pi_2$ and therefore final consumptions of the two goods $c_1 = \pi_1 l_1$ and $c_2 = \pi_2 l_2$ providing a modified CES utility

$$U = [\alpha_1(c_1 - \beta_1)^\rho + \alpha_2(c_2 - \beta_2)^\rho]^{\frac{1}{\rho}} \tag{1}$$

Maximising $U$ under the constraint $c_1/\pi_1 + c_2/\pi_2 = 1$ yields, for $= 1,2$ :

$$c_i - \beta_i = \left(1 - \frac{\beta_1}{\pi_1} - \frac{\beta_2}{\pi_2}\right) \frac{\alpha_i^{1/(1-\rho)} \pi_i^{1/(1-\rho)}}{\alpha_1^{1/(1-\rho)} \pi_1^{\rho/(1-\rho)} + \alpha_2^{1/(1-\rho)} \pi_2^{\rho/(1-\rho)}} \tag{2}$$

implying a global utility:

$$U(\pi_1, \pi_2) = \left(1 - \frac{\beta_1}{\pi_1} - \frac{\beta_2}{\pi_2}\right) \left(\alpha_1^{1/(1-\rho)} \pi_1^{\rho/(1-\rho)} + \alpha_2^{1/(1-\rho)} \pi_2^{\rho/(1-\rho)}\right)^{(1-\rho)/\rho} \tag{3}$$

This utility can also be written as a function of nominal income and equilibrium prices. With the linear technology used here, we have equilibrium prices that are independent of preferences and equal to $p_1 = R/\pi_1$ and $p_2 = R/\pi_2$. This implies, directly:

$$V(R, p_1, p_2) = (R - \beta_1 p_1 - \beta_2 p_2)\left(\alpha_1^{1/(1-\rho)} p_1^{-\rho/(1-\rho)} + \alpha_2^{1/(1-\rho)} p_2^{-\rho/(1-\rho)}\right)^{(1-\rho)/\rho} \tag{4}$$

Based on this result, the implementation of the equivalent income is straightforward. For given preferences, the equivalent income is the one which would be required to achieve the same utility as the utility $V(R, p_1, p_2)$ under the reference price system $(p_{1,ref}, p_{2,ref})$. It is therefore the solution to the equation $V\left(R^{eq}, p_{1,ref}, p_{2,ref}\right) = V(R, p_1, p_2)$ which gives:

$$R^{eq} = \beta_1 p_{1,ref} + \beta_2 p_{2,ref} + (R - \beta_1 p_1 - \beta_2 p_2)\left(\frac{\alpha_1^{1/(1-\rho)} p_1^{-\rho/(1-\rho)} + \alpha_2^{1/(1-\rho)} p_2^{-\rho/(1-\rho)}}{\alpha_1^{1/(1-\rho)} p_{1,ref}^{-\rho/(1-\rho)} + \alpha_2^{1/(1-\rho)} p_{2,ref}^{-\rho/(1-\rho)}}\right)^{(1-\rho)/\rho} \tag{5}$$

We can check that it is equal to current income when the reference prices are current prices.

Two scenarios will then be considered: a scenario where only productivities vary, with fixed preferences, and a scenario where the same variations in productivity and prices are accompanied by changes in preferences, which will only concern the parameter $\beta_2$. In principle, when preferences change, this expression (5) is calculated with the current parameters $\alpha_i$, $\beta_i$ and $\rho$, it is even its principle to do so. Only for prices is a time-independent reference structure needed.

Nevertheless, we will compare the results of this approach with the results of using terminal preferences, as proposed by Baqaee and Burstein (2021), or, conversely, relying on initial preferences.

The simulations assume the following values:

- A productivity $\pi_1$ growing linearly from 2 to 3 in 20 periods
- Productivity $\pi_2$ increasing from 2 to 3 over the first 10 periods, thus faster gains from t=1 to t=10 and then falling back linearly to 1.5 from period 11 to period 20 (e.g. a brown good that becomes more expensive again after having benefited from a sharp price drop in the first half of the simulation)
- $\rho$ constant equal to 1, hence and elasticity of substitution $\sigma = \frac{1}{1-\rho} = 0.5$
- $\alpha_1$ and $\alpha_2$ also constant at 0.25 et 0.75 respectively
- $\beta_1 = 1$ throughout the simulation period (i.e. an essential good whose consumption $c_1$ must always be at least equal to 1).

- $\beta_2 < 0$ (non−essential good) with the two variants: the fixed preference variant in which this parameter remains constantly equal to −1, and the variable preference variant in which this good 2 is judged to be increasingly superfluous as its price rises, with $\beta_2$ moving from −1 to −2 between periods 11 and 20.

Figures C1-A and B show the evolution of $l_i$, $c_i$ and $p_i$ in the two scenarios.

As far as prices are concerned, it was agreed to take the price of good 1 as the numeraire, and, as already indicated, the evolution of relative prices depends only on relative productivities, without any impact of changes in preferences: the evolution of the relative price $p_2$ is therefore the same in both cases. It varies like the ratio $\pi_1/\pi_2$ which goes from 1 to 2.5/3=0.833, then to 3/1.5=2 at the end of the simulation.

The distributions of activity and consumption are also the same from t=1 to t=10 since the preferences are the same in both cases, but they differ afterwards. Productivity progress on the essential good 1 allows a decreasing share of labour input to be allocated to it, and, with fixed preferences, this movement continues even when the productivity trend is reversed beyond t=10 in sector 2. On the other hand, if there is a disaffection for good 2 beyond t=10, there is a return of labour to sector 1. Consumption is reduced in both cases with the reversal of $\pi_2$ but more so in the scenario with disaffection for good 2.

What then about the implementation of the equivalent income, compared to what would be the result of calculating a chained price volume?

With unchanged preferences, the chained volume gives a result very close to the equivalent income evaluated by taking the initial prices as a reference. On the other hand, reference prices equal to final prices give a very high value to good 2, which leads to a much higher valuation of the increase in volumes between t=1 and t=10, which the subsequent reversal does not make up for.

With variable preferences, the volume at chained prices slows down in the second half of the projection, but without reversing the trend: as there is a disaffection for good 2, its chaining weight decreases, and the impact of its price increase on the income deflator in value terms is reduced, which allows the chained volume to return to growth, which is driven by the productivity progress that continues in sector 1. All the indicators reconvert at the end of the projection, but following a back and forth movement as regards equivalent income evaluated at terminal year prices, which continues to give stronger growth between t=1 and t=10, exactly the same as in the first graph.

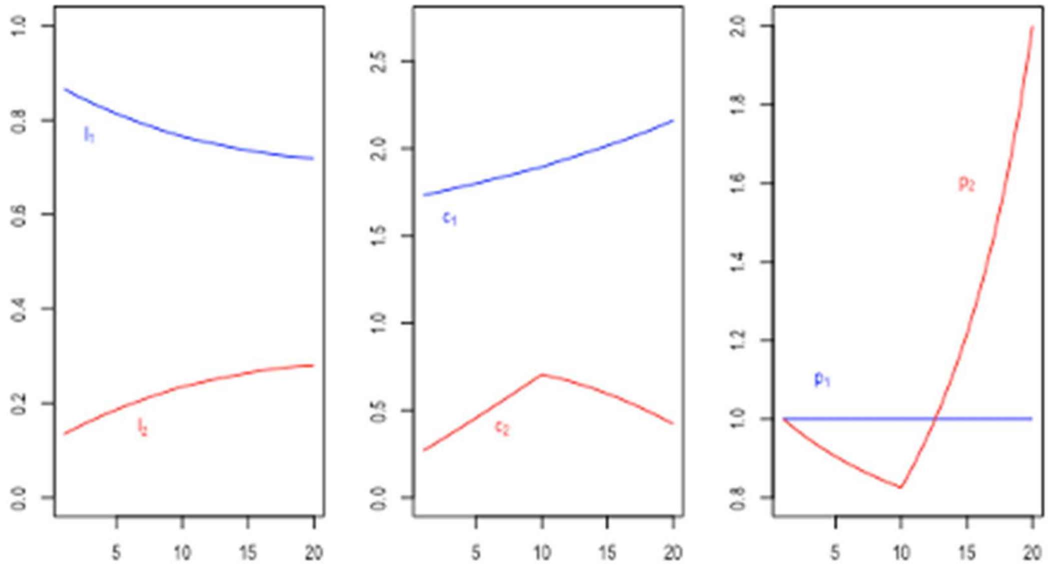
When the equivalent income uses the initial preferences, we find by construction the equivalent income evolutions of the scenario with fixed preferences. Prices and reference preferences are in fact the same in both cases. On the other hand, this time, it is the valuation at terminal year prices that converges with the volume at chained prices.

The opposite is true when terminal preferences are used: this time, it is the valuation at initial prices that ultimately converges with the estimate of the volume at chained prices. Combining valuation at final preferences and final prices leads to a much stronger estimate of growth throughout the simulation.

This first comparison shows the difficulty of having unambiguous messages when preferences change. Reasoning with current preferences has the advantage of reducing the field of possible evaluations, without having to choose between terminal and initial preferences. This is fully compatible with the principle of equivalent income: it allows in general to compare individuals with different preferences, and this is true whether the difference in preference structure is due to distance in time rather than in space or social hierarchy. The result reached remains nevertheless relative, depending upon the reference prices that are chosen.

Figure C1 – **Sectoral changes for activity, consumption and relative prices**

A – Scenario A : fixed preferences




B – Scenario B : variable preferences

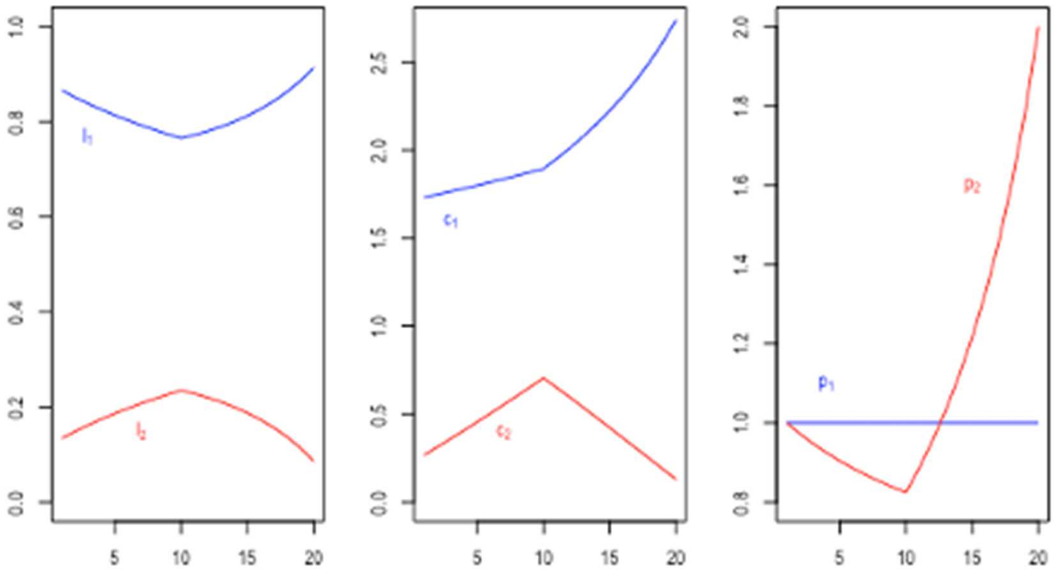


Figure C2 – **Chained volumes and equivalent incomes**