CHAPTER 2
THE GENERAL PRINCIPLES WHICH GUIDE THE COMPILATION OF THE QUARTERLY ACCOUNTS

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ESA 95: The methods used to draw up quarterly accounts fall into two major categories: direct procedures and indirect procedures. The use of direct procedures is dependent on the availability at quarterly intervals, naturally requiring certain simplifications, of the same data sources used to compile the annual accounts. On the contrary, indirect procedures involve breaking down annual data using mathematical and statistical techniques and drawing upon reference indicators which allow to make extrapolations for the current year."

The French quarterly accounts are constructed using an indirect method,[2] based on two major data series: the annual national accounts, and the short-term outlook data compiled from various sources.

The annual accounts are an exhaustive collection of economic data, or at least as exhaustive as it is possible to be, summarising this data within the framework of the national accounts. These data are of a superior quality, but the downside of this quality is the time it takes to compile and publish the accounts. The economic situation for a given year (Y) is thus described for the first time in the annual accounts published the following May; these are the provisional accounts, compiled largely with the help of incomplete, short-term information.[3] One year later, i.e. in May of the year Y+2, the accounts for year Y are revised and upgraded to the status of 'semi-definitive'; they are still only partly based on exhaustive data. The definitive version of the accounts for year Y is finally published in May of year Y+3.

Conversely, various sources of short-term information, published monthly or quarterly, are rapidly available and offer an immediate overview of recent economic performance. This information is published by the INSEE and the various other agencies charged with compiling statistical information (the General Directorate for Public Finances, the Banque de France, etc.). These sources often have to strike a difficult balance between the speed of publication and the quality of the statistics involved, which may for example mean that their statistical construction is based on samples. In such cases, as far as data from companies is concerned, the information will not cover recently-founded businesses and is thus likely to cause a cyclical divergence with regard to the exhaustive annual information, if the demographics of business are linked to this cycle.

These short-term data sources often diverge, in terms of their level and their evolution, from the annual national accounts, largely due to differences in terms of definition and scope. For example, the turnover indices used to measure industrial output in certain sectors do not correspond precisely to the definition of output used in the national accounts, as they only take sales into account and do not include inventories. Variations in inventory levels can thus partly explain the differences observed between the annual accounts' output figures and the annualised turnover indices. It may also be the case that short-term data do not correspond precisely to the scopes defined in the national accounts. Thus the salaried labour figures issued quarterly by the INSEE are divided into various sectors of activity, whereas the employment data contained in the national accounts are broken down into branches of activity.[4] The evolution of the salaried labour index thus only partially reflects the development of employment levels per branch, failing to distinguish between the various different activities which may be represented within a single company.

The quarterly accounts ensure a certain coherency between the exhaustive annual data and the more rapidly-available, but less complete, short-term information. The methodology used to reconcile these two sources is largely dependent on a process of calibration and fitting:

[2] A number of other EU countries also use such indirect methods, but in the vast majority of cases quarterly accounts are compiled using direct methods.

[3] This provisional account is actually jointly compiled by the accountants responsible for the annual and the quarterly accounts.

[4] The salaried labour force for a given branch of activity covers all workers performing the activity in question, regardless of the nature of their employer, while the labour force for a given sector of activity counts the total number of employees attached to companies whose principal activity lies within that sector.

- calibration transforms the short-term data, making them conform to the definition and scope of the corresponding data series in the annual accounts;
- fitting subsequently ensures complete coherency between the data series used in the quarterly accounts and the annual accounts.

2.1. The twin pillars of the method: calibration and fitting

2.1.1. Calibration

The quarterly accounts attribute a monthly or quarterly **indicator** to each item in the national accounts, rapidly available and corresponding as closely as possible to the concept and scope of this item. This association is achieved by creating an intermediary aggregation: for example automobile production, export of agricultural products etc. The fundamental purpose of the quarterly accounts is to 'adapt' the indicators to fit the annual accounts: by estimating the historical statistical relationship between the annual-adjusted indicator and its corresponding item in the definitive results, and by postulating that this relationship derived from the annual data will also be applicable to the quarterly data.

The statistical method used to construct the quarterly accounts thus seeks to correct the systematic differences which exist between the information contained in the short-term indicators and that provided by the annual accounts. For example, when estimating manufacturing output the indicators used in the majority of branches are the industrial production indices (IPI). But certain branches are not fully covered by the classification at the most detailed level (size criteria, in particular, mean that the smallest businesses are excluded). By way of an example, if we consider that, for a given branch, the output from businesses not covered by the survey (very small companies, for instance) is growing at a faster average rate than the output from other businesses, the trend shown by the IPI will actually be an underestimation of the actual annual output of this branch. In such cases we can use a statistical model to correct the bias by upwardly revising the growth figures provided by the IPI.

The calibration equation is a simple linear equation linking the annual accounts to the annual-adjusted indicator, which can be expressed as follows for a given year a :

$$\boxed{C_a = \alpha + \beta \times I_a + u_a}$$

where C_a is the annual account for year a, I_a represents the annualised indicator, i.e. the annual sum of the quarterly indicators: $I_a = \sum_{t=1}^{4} I_{a,t}$, and u_a is the residual of the calibration model, representing the developments of this data item which are not reflected in the evolution of the benchmark indicator.

The coefficients α and β are estimated using the data item and the indicator over several years, covering a long enough period to guarantee the statistical accuracy of the estimate. Three models are used to optimise this estimate, depending on the statistical properties of the calibration residual.[5] We can thus use ε_a to represent the residual element in these models, which should be white noise if the model has been well chosen.[6]

[5] On the importance of the statistical properties of this residual, see Bournay & Laroque [5].

[6] Conceptually, this method is thus dependent on the statistical nature of the annual relationship between the accounts and the indicator. This process differs from other commonly-used temporal disaggregation methods, such as the Chow & Lin model [8]. Such methods first create a model representing the statistical properties of the quarterly relationship, which is then estimated based on the annual figures.

- The estimate is derived from the level model, whenever the calibration residual is stationary and not autocorrelated. The following equation is calculated using the ordinary least squares method (OLS):

$$C_a = \alpha + \beta \times I_a + \varepsilon_a$$

The calibration residual u_a is thus equal to the residual of the estimate ε_a.

- The estimate is based on the level model, whenever the calibration residual introduces an element of autocorrelation. The equation uses quasi-least squares (QLS):

$$C_a = \alpha + \beta \times I_a + u_a$$

based on the hypothesis that $u_a = \rho \times u_{a-1} + \varepsilon_a$

This model offers two advantages:

- when it comes to estimating the coefficients, if it is the ‘true model’, the QLS method will give a more efficient estimate than the OLS alternative;
- when it comes to making predictions for the annual accounts, modelling the evolution of the calibration residual allows to assess its persistence and apply this to the estimates for the current year.

- The model is estimated in terms of differences, whenever the calibration residual is not stationary. This model is a more extreme version of the previous one, with ρ =1. The following equation uses ordinary least squares:

$$\Delta C_a = \gamma + \beta \times \Delta I_a + \varepsilon_a$$

The calibration residual u_a can thus be expressed as $u_a = u_{a-1} + \varepsilon_a$.

In this case, if the coefficient γ is significantly above or below zero, the calibration equation linking the indicator to the accounts is actually:

$$C_a = \beta \times I_a + \gamma \times T_a + u_a$$

where T_a is a linear trend and u_a does not necessarily present a zero mean.

This model again has two advantages:

- if the annual accounts and the indicator are integrated series of order 1, and not cointegrated, their differencing allows to work with stationary series and perform standard tests;
- as with the previous model, taking the persistence of the calibration residual into account allows to refine the estimate of the account for the current year. The model can be considered highly plausible if a divergence between the account and the indicator in a given year opens up a permanent gap between the two series.

An example of a choice of a calibration model is given in *Appendix 1*. This choice is made using different statistical criteria. The portmanteau test (*Appendix 2*) allows to rule out any models for which the residual of the estimated equation is autocorrelated. The standard deviation of the residual allows to compare the explanatory potential of the three methods across the whole period, while the standard deviation of the recursive residuals allows for comparison of the predictive capacity.

This relationship between the accounts and the annualised indicator is held to be stable so that, when estimated over the past, it continues to be applicable for the very recent past, and also allows to construct the best possible predictions for the years for which the definitive annual accounts have not yet been released. Furthermore the indicator bias, corrected by the calibration equation, is supposed to be constant over the course of the year. In this way, the same equation can be used to 'correct' the quarterly indicator. The estimated coefficients α and β are then applied to this indicator and the first estimate (before fitting) obtained for the quarterly account will correspond to:

$$C_{a,t} = \frac{\alpha}{4} + \beta \times I_{a,t}$$

where $C_{a,t}$ represents the account total for the quarter t in the year a.[7]

2.1.2. Fitting

Simply applying the estimated relationship between the indicator and the annual accounts does not allow to preserve at a quarterly frequency all of the information contained in the annual accounts. The calibration factor, which remains, contains information that even the adjusted indicator cannot provide. This information thus needs to be preserved by 'fitting' the quarterly accounts to the annual accounts for all available past annual exercises. To do this, the annual calibration residual needs to be split across each quarter of the year in question.

In theory, this practice of 'quarterly breakdowns' of calibration residuals should take into account the statistical properties of the calibration residual, with different approaches for autocorrelated, non-autocorrelated and non-stationary residuals. Simply dividing the annual calibration factor by four, in theory the most suitable approach if the residual is not autocorrelated, is not in fact the solution adopted. A simple division can indeed generate substantial changes in level in certain first quarters, if for the year concerned the calibration residual turns out to be relatively significant compared to the annual account. In practice the 'quarterly breakdown' method used, which splits the calibration residual over these four quarters, has been developed for reasons of pragmatism, allowing to minimise variation from one quarter to the next so that the calibration residual has as limited an effect as possible on the volatility of the quarterly accounts. This 'smoothing' method ensures that the quarterly evolution of the calibration factor is as regular as possible, and to do so uses a system which minimises the sum of the squared deviation observed between successive calibration factor values (cf. section 2.1.4 on smoothing).

The quarterly account thus becomes:

$$C_{a,t} = \frac{\alpha}{4} + \beta \times I_{a,t} + u_{a,t}$$

where $u_{a,t}$ is the quarterly calibration residual for quarter t in year a, obtained by smoothing the annual calibration residual u_a.

[7] If the model has been estimated based on differences and the constant is significant, the calibration relation will demonstrate a linear trend which must be reflected in the quarterly data, and as such the equation becomes:

$$C_{a,t} = \beta \times I_{a,t} + \gamma \times T_{a,t}$$

where $T_{a,t}$ is a linear trend such that $\sum_{t=1}^{4} T_{a,t} = T_a$ for all years a. For example, the trend $T_{a,t}$ can be defined as: $T_{a,t} = (4 \times a + t)/16$ (for year a and quarter t, ranging from 1 to 4) and the annual trend T_a must then be defined as: $T_a = (16 \times a + 10)/16$.

The quarterly calibration residual is calculated to ensure that for each year it respects the formula $\sum_{t=1}^{4} u_{a,t} = u_a$ [8].

For each year up to and including the year covered by the provisional account, the sum of the quarterly accounts for the year should be equal to the total value of the annual account. For the current year, which has not yet been 'fitted', the annual calibration factor can be extrapolated. This extrapolation is based on the assumption that the residuals estimated by the models are white noise; this residual is thus considered null for the year at hand. As a result, the extrapolation method used to calculate the calibration residual will depend on the model used to produce the estimates.

For the level model, the calibration residual is not auto-correlated and so:

$$\boxed{u_{P+1} = 0}$$

where P represents the year covered by the provisional account.

If the calibration residual is autocorrelated but stationary,

$$\boxed{u_{P+1} = \hat{\rho} u_P}$$

If the calibration residual is non-stationary,

$$\boxed{u_{P+1} = u_P}$$

The smoothing procedure is applied after this extrapolation of the annual calibration residuals, ensuring that there is no abrupt variation between the fitted quarters and the quarters which correspond to the provisional account.

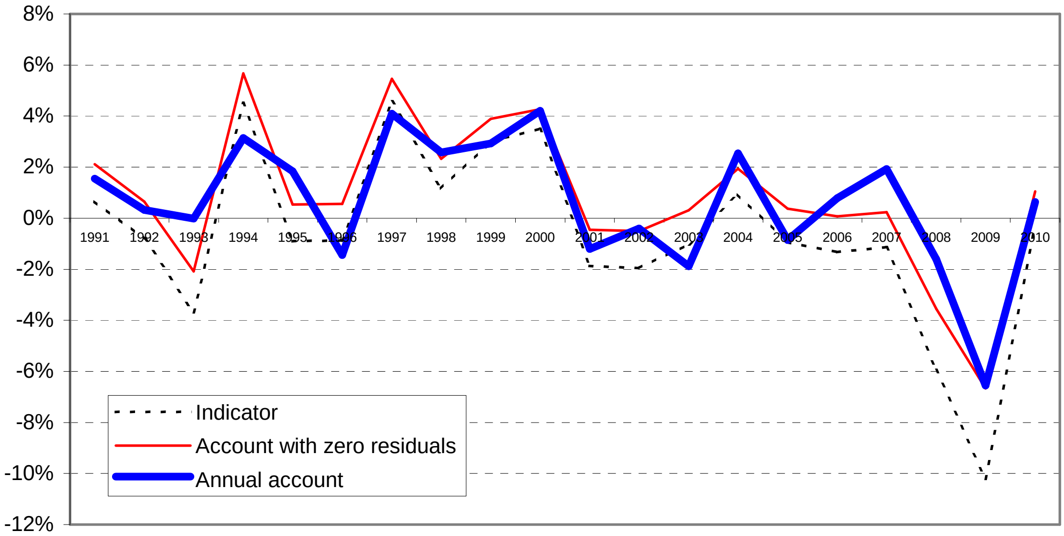
[8] If the model has been estimated based on differences and the constant is significant, this becomes:

$$C_{a,t} = \beta \times I_{a,t} + \gamma \times T_{a,t} + u_{a,t}$$

2.1.3. Example

This method of calibration based on the annual data and fitting via quarterly smoothing of the residuals is illustrated with the example of wood and paper production. *Graph 1* compares the growth rates of the annual account (C_a), the indicator (I_a) and the account with zero residuals. This last figure corresponds to the results of the calibration, as they would have been if the annual account for the year in question was not known, that is to say if the calibration residual had been extended year-on-year so as to eliminate the residual.

Graph 1: Effects of calibration-fitting, illustration using the output figures for the wood-paper branch (indicator: IPI, estimation period: 1990-2009)

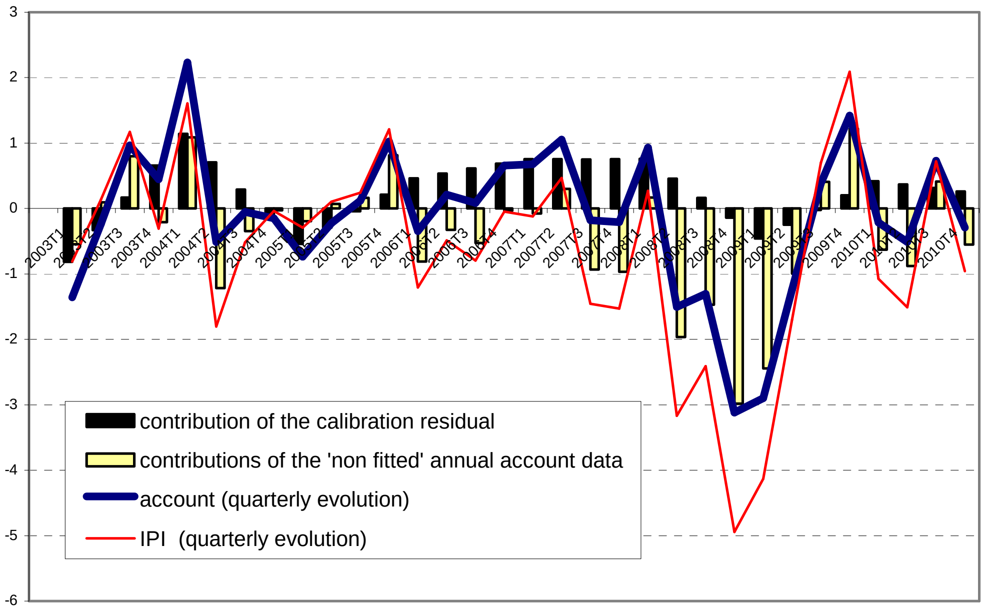


Source: quarterly national accounts, 2005 base

The output of the wood and paper branch is calibrated on the basis of the industrial production index (IPI). The progression of this indicator corresponds relatively closely to that of the annual account, but appears to be less dynamic on average over the whole period. The calibration process thus modifies the indicator to correct this bias, somewhat smoothing the progression but preserving the contours of the indicator's curve. Thus the zero residual account data displays a similar progression to that of the indicator, although more dynamic on average and closer to the annual account. The residuals are relatively small in comparison to the amplitude of the fluctuations seen in the accounts, which suggests that the IPI is a fairly good indicator of output.

Focusing on a shorter period for greater clarity, *Graph 2* illustrates the results of fitting by taking the same statistic (output of the wood-paper branch) and presenting the quarterly evolution of this figure after fitting, demonstrating the respective contributions of the 'non-fitted' annual account data ('zero residual') and the calibration residual. We can observe that the impact of the calibration residual on the volatility of the account data is kept to a bare minimum, bearing in mind the constraints involved in fitting the quarterly accounts to the annual accounts.

Graph 2: Effects of calibration-fitting shown in quarterly terms, illustrated using the output figures for the wood-paper activity branch



Source: quarterly national accounts, 2005 base

2.1.4. Smoothing

Smoothing is used in two main situations. As explained in section 2.1.2, it can be used to accurately divide the annual calibration factors obtained from the calibration process and to thus ensure coherency between the quarterly and annual accounts. Smoothing also allows to achieve quarterly breakdowns of certain data series for which no short-term figures are available: annual accounts are projected via a process of extrapolation to cover the current year, and quarterly accounts are then derived directly from the smoothing of the annual data.

This requires an extrapolation of the annual account which is not yet available into the current year. This operation is generally conducted on the basis of *ad hoc* judgements (such as those provided by the accounts committees or by INSEE experts in specific fields) or, where necessary, by a ‘nuanced’ projection based on past trends.

The procedure used to smooth an annual series C_a, for use in the quarterly accounts, consists of producing estimates for a quarterly series $C_{a,t}$ minimising the sum of the squared deviations from one quarter to the next while observing the constraint that, for each year a, the sum of all quarters must be equal to the annual total C_a:

$$\min \sum_t (C_{a,t} - C_{a,t-1})^2$$

$$\text{with the constraint that : } \sum_{t=1}^{4} C_{a,t} = C_a \text{ for all years } a$$

This squared deviation technique is discussed in *Appendix 3*.

2.1.5. An intermediate method between smoothing and calibration: rate smoothing

The calibration-fitting process can be used to produce a quarterly account breakdown, as long as we have an indicator which adheres sufficiently closely to the definition and scope of the corresponding item in the annual accounts. If no such indicator is available, a quarterly breakdown is often obtained by smoothing the annual account data. Nonetheless, in certain cases there may be an indirect indicator which is linked to a particular account, notably via a rate of taxation or payments. For example, no short-term information is available regarding the taxes paid on imports. Nonetheless, these taxes are linked to imports by the applicable tax rate. The calibration method is not applicable in this case, as that would imply a linear relationship between the two accounts, i.e. a constant nominal rate of taxation. When considering the current year, it is impossible to base calculations on the assumption that the nominal rate will remain constant if tax rates are set to change.

A more suitable method is therefore used, which consists in smoothing the ratio between these two data series, i.e. the nominal tax rate in this case. As with simple smoothing, this method requires to extrapolate figures for the current year, allowing to take developments in the legislation and regulations into account.

This method of rate smoothing is also useful for assessing the value of an accounting item based on volume, if no price indicator is available: the process is then equivalent to smoothing the annual price.

Let us consider the example of an annual account T_a (customs duties, to continue the example given above) which is economically associated with another annual account C_a (imports, for the purposes of our example). They are linked via a rate, tx_a. If no quarterly information is available regarding this rate, or even directly regarding accounting item T, the ‘rate smoothing’ procedure involves estimating the quarterly rate $tx_{a,t}$ by minimising the squared deviations between quarters, with the constraint that for each year a, the sum of the quarters $T_{a,t}$ must be equal to the annual total:

$$\min \sum_t (tx_{a,t} - tx_{a,t-1})^2$$

$$\text{with the constraint that : } \sum_{t=1}^{4} tx_{a,t} \times C_{a,t} = tx_a \times C_a = T_a \text{ for all years } a$$

The quarterly account is thus defined by the equation:

$$T_{a,t} = tx_{a,t} \times C_{a,t}$$

The minimisation of squared deviations is explained in *Appendix 4*.

2.2. The working day adjustment (WDA) and seasonal adjustment (SA) procedures

The aim of the quarterly accounts is to retrace the developments of the major economic variables at a quarterly rate. However, the variations in the raw account data from one quarter to the next can be difficult to interpret due to the impact of seasonal effects and the variation in the number of working days in each quarter.

To make the quarterly accounts easier to read, the data series are corrected for the effect of the variation in the number of working days (WDA) and the variation between seasons (SA). These corrections are applied to the indicators, then a process of calibration and fitting allows to produce the 'SA-WDA' account.[9]

In practice, the coefficients which make up the calibration equation and residuals are estimated on the basis of the raw indicators and the annual accounts. The raw quarterly account is obtained by applying the calibration-fitting process to the raw indicator:

$$C_{a,t}^{raw} = \frac{\alpha}{4} + \beta \times I_{a,t}^{raw} + u_{a,t}$$

whereas the SA-WDA quarterly account is derived from the SA-WDA indicator, in application of the parameters calculated using the raw data:

$$C_{a,t}^{SA-WDA} = \frac{\alpha}{4} + \beta \times I_{a,t}^{SA-WDA} + u_{a,t}$$

An alternative method would be to calculate the raw account total using the calibration-fitting process, then to apply the statistical techniques directly to this raw account and not the indicators. This method would not allow to perform certain seasonal adjustment operations at more precise levels of detail than those already possible with the calibration model. And yet such precise seasonal adjustment can be very useful when an account is calibrated on the basis of a sum of indicators corresponding to heterogeneous products which have not the same seasonal effects.

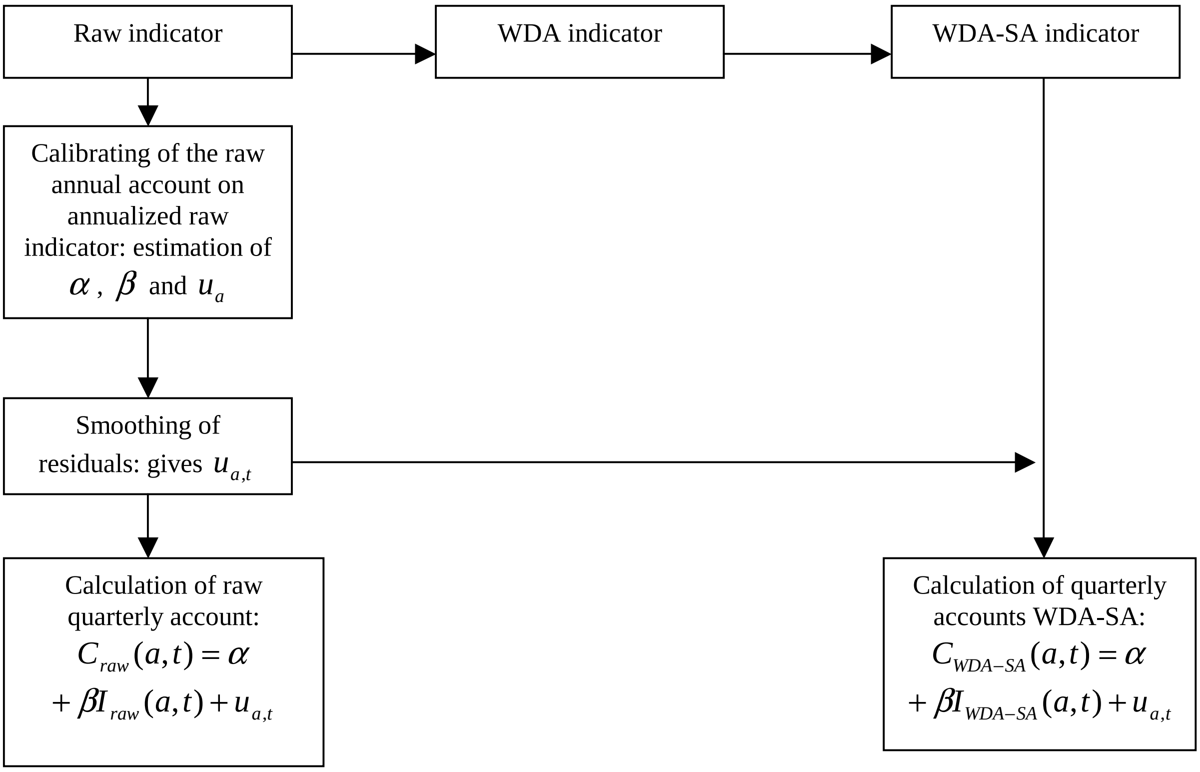
It should also be noted that the method used by the quarterly accounts requires seasonal variation to be neutral across the year as a whole. This is a customary practice which makes the annual data series easier to read and avoids deforming those series whose seasonal variation is stable from one year to the next.

On the other hand, the correction for the variation in number of working days is not neutral at the annual level: the number of public holidays, for example, varies from year to year and thus affects the variation from one year to the next. The annual total of an SA-WDA indicator is thus different from the total of the raw indicator: the difference corresponds to the annual effect of the correction for working day variation. And yet the same calibration coefficients and residuals are applied to obtain both the raw quarterly accounts and the SA-WDA figures. The accumulated annual total of the quarterly SA-WDA accounts is thus different from the total raw value of the account, i.e. the annual account: this total is in fact equivalent to the value of the annual account adjusted for the effects of working day variation.

The general method used to turn a raw indicator into a quarterly SA-WDA account is outlined in *Diagram 1*.

[9] The abbreviation SA-WDA appears frequently in various INSEE publications and information tables. It does not, however, reflect the chronological order of the process, since the indicators are first corrected for the effect of working day fluctuation and then subsequently adjusted for seasonal variations.

Diagram 1: Method used to derive the quarterly SA-WDA account from a raw indicator



Source: quarterly national accounts

2.2.1. Correcting the effect of the variation in the number of working days

The variation in the number of working days can potentially have a very significant impact on economic data series. To give an extreme example, if output was entirely proportional to the number of hours worked then the output of a given quarter containing a public holiday would be around 1.5% lower than the output of the previous quarter which had no public holidays (i.e. 1/65, with 65 the average number of working days in a quarter with no public holidays). It is important to ensure that such reductions are not confused with effects related to the economic cycle. The purpose of correcting working day variation is thus to construct data series which reflect ‘identical numbers of working days’, allowing for analysis of economic developments which are not corrupted by the differences in number of working days from one quarter to the next.

In fact, the effects of working day variation are far from proportional. For instance, production processes are adaptable and can allow firms to partly compensate for the output lost due to a public holiday. Moreover the effects can be different depending on the day of the week: on the one hand, employee productivity is not the same for each day of the week, and on the other hand various forms of consumption perform more strongly on Saturdays than on other days of the week.

All of which militates in favour of statistical methods which measure the effect of the number of working days for each data series, with the capacity to differentiate between different days of the week. These methods are generally more efficient when applied to monthly data series than they are with quarterly figures. As it happens, the number of public holidays and the number of each day of the week included in a quarter do not vary substantially from one year to the next,[10] which makes it difficult to estimate what effect such variation might have. On the contrary, fluctuations in the number of public holidays are more significant when seen on a monthly scale. Thus for the quarterly accounts, and in so far as is possible, working day adjustment is performed using monthly indicators.

[10] For example, the 2nd quarter contains precisely 13 weeks, with 13 Mondays, 13 Tuesdays… 13 Sundays.

Working day adjustment is based on the assumption that the evolution of a statistical indicator can be broken down into two orthogonal (non-correlated) components: one component linked purely to the effects of the number of working days, and a second component adjusted for working days (WDA), which also incorporates an adjustment for the seasonal circumstances of the data series. The component corresponding to the effect of the number of working days should thus be estimated independently of all seasonal considerations; the average effect of the public holidays associated with a particular month (e.g. Christmas) is incorporated into the seasonal component.

The basic method used for the quarterly accounts is simple: a regression of the raw monthly variable using variables representing, in order, the number of Mondays through to Saturdays worked (i.e. not public holidays) and the number of Sundays, public holidays or not, in each month. So as to avoid incorporating the effects of seasonal variation, these variables are seasonally adjusted, retaining only the average deviation from the mean for each month. The sum of all working days, Sundays and public holidays not falling on a Sunday should be equal to the total number of days in the month, which is always the same except for in February, which is the only month to have a variable number of days (28 or 29). We thus need to add a leap year indicator (cf. *Appendix 5* for more details).

The raw indicator is thus broken down as follows:

$$I_t = \alpha_1 N_{Mon}^{work} + \alpha_2 N_{Tue}^{work} + ... + \alpha_6 N_{Sat}^{work} + \alpha_7 N_{Sun} + \beta I_{leap} + I_t^{WDA}$$

where N_{Mon}^{work} is the number of Mondays worked in month t adjusted for seasonal variation, I_{leap} is the leap year indicator and I_t^{WDA}, the residual of this equation, is the indicator adjusted for working days.

In this model, the effect of public holidays cannot be detected. As the sum of all working days, Sundays (holidays or not) and public holidays (excluding those falling on Sundays) is constant for each month from one year to the next (except February in leap years), the coefficients estimated for the adjustment of working day variation should be interpreted with reference to the number of public holidays. As such the coefficient for working Mondays does not correspond to the gross effect of a working Monday, but to the effect of this day relative to the effect of a public holiday. For example, household expenditure on "Accomodation and food services" linked to tourist activities is greater on public holidays than it is on working days. The coefficients attached to the number of working days are thus negative.

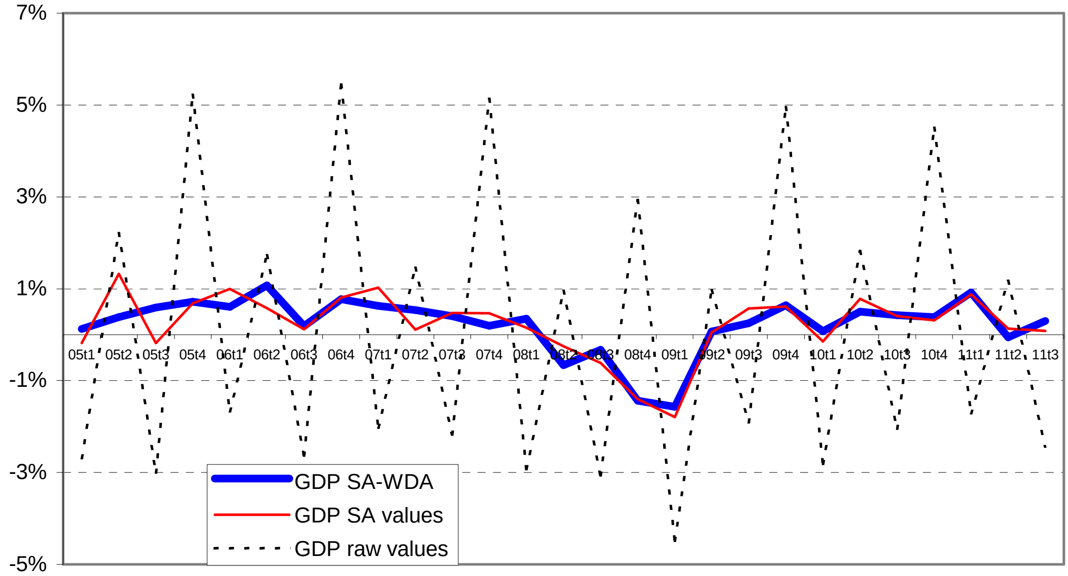
In practice the statistical work required to estimate the effect of working days is split into several successive stages, including a certain number of tests. These steps and the overall methodology are set out clearly in *Appendix 5*.

The choice of models and the estimation of the coefficients are reviewed every year, at the same time as the provisional accounts for the past year are being drawn up.

By way of an illustration, the absolute effect of working day variation currently represents between 0.1 and 0.5 GDP growth points quarterly (*Graph 3),* and 0.1 and 0.2 points annually. The effect of WDA on GDP is less dramatic than the effect of SA, but WDA can sometimes have a significant influence on the calculation of quarterly growth estimates.

To be more specific, GDP growth expressed in seasonally-adjusted terms (without WDA) for Q3 2011 was +0.1%, while the same figure after SA and WDA was +0.3%, the latter figure being the one used when estimating the detailed results for Q3 2011. The impact of working day adjustment on the rate of GDP growth was thus +0.2 points in the third quarter, up from -0.2 points in the second quarter. This negative effect in Q2 has a simple explanation: unusually, the month of May 2011 contained no public holidays, and thus more working days, as both Mayday and 8th May fell on Sundays.

Graph 3: Quarterly Gross Domestic Product expressed in chained volumes, raw values, SA values and SA-WDA values



Source: quarterly national accounts, 2005 base, 3rd quarter of 2011 detailed results publication.

2.2.2. Seasonal adjustment

The majority of items recorded in the national accounts (output, consumption) display marked seasonal variations. Economic output, for example, is less dynamic in July and August when many firms reduce their activity as a result of the summer holidays. Energy consumption is much higher during the winter months, when heating expenses are added to other, year-round uses. The quarterly growth of GDP can fluctuate significantly, as illustrated in *Graph 3*, and displays regular seasonal patterns: GDP is systematically lower during the summer months, but we should not conclude from this that the overall economic outlook is any weaker.

To fully understand the underlying short-term developments, we need a set of measurements which are not subject to seasonal variation.

In the quarterly accounts, the effects of seasonal variation are estimated on the basis of the indicators, which are already adjusted for working day variation. Two methods of seasonal adjustment are used. The first, known as the Buys-Ballot method, is a standard deviation method. The process involves assuming the seasonal variation for a given quarter to be equal to the mean value of the series observed for this quarter, in all years for which data are available, less the total mean (*Appendix 6).* This is a relatively unsophisticated method which takes no account of evolutions in seasonal variation. As such the method is only used for very short data series, for example in cases where an indicator displays unusual seasonal variation in a given year and needs to undergo separate adjustment in two distinct periods.

The second method of seasonal adjustment is that employed by the software programme 'X12-ARIMA' (*Appendix 7*). This method is based on a series of moving averages applied to the data series. These moving averages allow to extract the seasonal components, but first require to extrapolate the series over several months. This extrapolation is performed using ARIMA-type models.

The seasonal adjustment variation for a given indicator is constantly re-estimated, each time new data is available for the most recent periods. These new figures lead to frequent revisions. The type of calculation method (based on addition or multiplication) and the ARIMA models which are used to extrapolate data series before estimating seasonal variation are reviewed every year, when the time comes to draw up the provisional accounts for the past year.

2.3. Volumes at last year's constant average prices versus volumes at constant prices

One of the major objectives of the national accounts is to describe the changes in the **volumes** expressed in the major economic aggregates, cancelling out the effects of price variation to analyse the growth of raw domestic output, consumption, etc. These volumes give a clearer idea of quantity. However, simply adding up the quantities of elementary components involved is pointless: the quantity of cars consumed is not directly comparable with the quantity of bicycles consumed. These quantities need to be commensurate, which can be achieved by linking elementary quantities with the prices they commanded in a given period of time.

The values are then comparable: the value of an aggregate is obtained by adding up the values of the elementary components involved, determined for each period on the basis of the quantity produced and the average price observed during this period. The volume and price of an aggregate are thus defined symmetrically: the price is the ratio of aggregated value to volume.

Calculating the volume of an aggregate requires to weight the volumes of its component elements based on prices. The choice of a reference period, which will determine the structure of the prices used to produce a weighted ranking of the different volumes, is thus of crucial importance. There are two options available:

- **calculating volumes using constant prices derived from the base year** (the slightly deceptive abbreviation 'constant volumes' is often used); in 2011, for example, this involved weighting the elementary levels which make up aggregates (sources of output, or branches of the economy) on the basis of their relative prices in the base year (in the '2005 base', the reference or 'base' year is 2005);
- **calculating volumes using the prices observed in the previous year**; in 2011, the sub-levels of aggregates were weighted using their most recent relative prices, i.e. the average prices recorded in the accounts for the year 2010.

By way of an example, in order to calculate the growth in total consumption in volume between 2010 and 2011, using the system of volumes based on last year's prices would lead to weight the 2011 evolution of the volume of electronic goods consumption based on the relative prices recorded in 2010, prices which are lower than those recorded for 2005 due to the significant price decreases seen between 2005 and 2010. If the consumption of electronic goods was dynamic in 2011, total consumption nonetheless appears less dynamic when calculated using the previous year's prices than it would do if calculated using the prices observed in the base year. This problem becomes progressively more significant the further we move from the base year.

The annual national accounts are themselves calculated and published in volumes based on the **chained** prices of the previous year (another somewhat misleading abbreviation means that this system is often referred to as the 'chained volumes' method), that is to say linked back to a specific reference year (the base year). The idea behind these chained indices is to preserve the evolutions, rather than the levels, of volumes at last year's prices from one year to the next, and to chain-link these evolutions starting from the values established for a given reference year.

Levels of volumes based on non-chained previous year prices cannot be used to construct time data series, since the change in volumes between two consecutive years must include both the change in prices (between two reference years) and the change in volumes.

Estimating volumes based on last year's chained prices thus offers the dual advantage of providing data suitable for constructing time series and also detecting any changes in the relative price structure: put simply, they provide a more satisfactory description of the economic reality when the prices of certain products evolve very differently to other prices – as is notably the case with new technologies. However, these chained volumes do pose certain problems:

- They can prove misleading when prices tend to oscillate rather than evolve following a coherent trend. This may be the case, for example, with agricultural prices and energy prices.[11]

[11] On this subject, see for example the analysis conducted by J-P. Berthier [4].

- Furthermore, these volumes lose their additivity with regard to volumes calculated at non-chained previous year prices and constant volumes. As such the uses-supply accounting balance is no longer respected in levels and the aggregates cannot be obtained using the same direct method employed as the sum of lower levels. Appendix 8 explains why this is not possible with chained volumes. The lack of additivity makes drawing up and publishing the accounts a more complex operation. Furthermore, this may cause certain problems for some users operating within the accounting framework: in the macro-econometric models, for example, the accounting framework is proffered to vouch for the coherency of the forecasts.

From a practical standpoint, measuring chained volumes is more complicated for the quarterly accounts than it is for the annual accounts. Thus, for sub-annual frequencies, several chain-linking techniques may be used.[12]

To obtain a rapid overview of structural price changes, it is possible to chain-link volumes to the prices of the previous quarter. This method is discouraged by both SNA 93 [20] and Eurostat [9]. It can cause distortions because infra-annual data are more subject to fluctuations than annual data, particularly due to seasonal effects.

It is therefore advisable to perform chain-linking with reference to an annual base price. There are at least three different ways of doing this:

- The method which most closely resembles the annual chain-linking process sees the quarterly accounts valued at the constant average prices of the previous year ('annual overlap'). This gives rise to a methodological discontinuity (which need not necessarily be substantial in empirical terms) affecting the first quarters: the change in volumes calculated for the first quarter of each year will reflect both a change in volume and a change in price, since the figure for the fourth quarter of the previous year was calculated using a different reference price. The scale of this 'chaining effect' on the evolution of volumes observed in the first quarter of each year will depend on the way inflation is spread across the different components. The advantage of this method is that it allows to preserve the additivity of the infra-annual data: the sum total of the four raw quarterly accounts is equal to the annual total for that year.
- To resolve this methodological discontinuity affecting the first quarter, another method exists: it involves calculating the volumes for the fourth quarter of year Y using both the average prices from the previous year and the prices recorded so far that year, so that the transition from the fourth quarter of year Y to the first quarter of year Y+1 corresponds to the growth in volumes calculated at year Y prices (one-quarter overlap). This method, however, does not preserve the sub-annual additivity of the quarterly figures.
- Another method sees the links in the chain in year-on-year terms, i.e. as a shift between the first quarter of the year Y+1 calculated at the annual average price of year Y and the first quarter of year Y also calculated at the annual average price of year Y (the over-the-year technique). The IMF manual advises against using this latter method [12], which does nothing to resolve the discontinuity and introduces a greater volatility.

12. All of these techniques are described in detail in the IMF's quarterly account manual [12] and by Arnaud F. [1].

2.4. Calculating volumes in the quarterly accounts

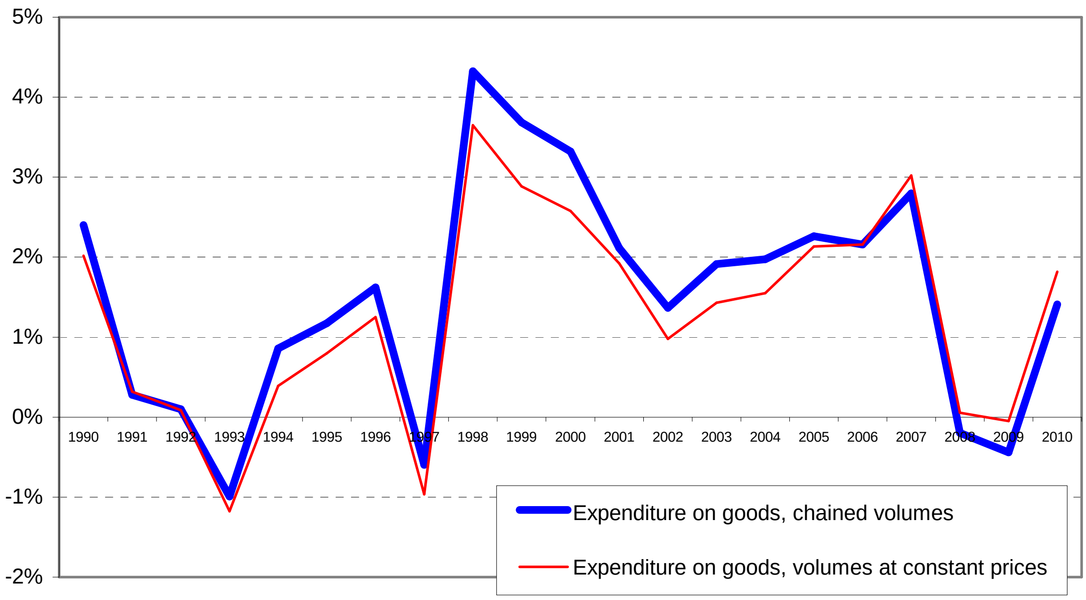
2.4.1. Accounts published with chained values…

Faced with these complex problems, until 2006 the French quarterly accounts used volumes calculated at constant prices derived from the base year. One of the arguments used to justify this system was that, in France, the difference between chain-linked volumes and the volumes obtained by using constant base-year prices was relatively insignificant. This was partly due to the fact that up until this period products based on new technologies, whose prices are liable to decrease rapidly, did not occupy a particularly important place in the French economy.

This argument became increasingly untenable as new technologies became ever more important (*Graph 4*). Applying the 2005 base figures to the year 2010, the deviation between the annual growth rate of household consumption on goods (an aggregate indicator published monthly by the INSEE) expressed in chained volumes, and the same figure expressed in constant prices from the base year, reaches 0.4 points.

A desire to harmonise the methods used to produce quarterly accounts throughout Europe, spurred on by the increasing disparity observed between the chained volumes and the volumes calculated at base-year prices for certain product-operation pairs, led to a rethink: since 2007 the French quarterly accounts have been published in volumes chain-linked to the previous year's prices, using the annual overlap method. The principal advantage of this method is that it allows to retain infra-annual additivity.

Graph 4: Year-on-year evolution of household expenditure on goods – a comparison of chained volumes and volumes calculated at constant prices based on the reference year 2005



Source: national accounts, 2005 base, WDA-SA data (3rd quarter of 2011 detailed figures publication)

2.4.2. …but constructed based on volumes estimated at constant prices derived from the base year

Although they do present ‘chained volumes’, the quarterly accounts are nonetheless still calculated using constant prices derived from the reference or ‘base’ year (‘constant volumes’).

When constructing the quarterly accounts, the supply-uses balances (SUB) are obtained by balancing certain operations. For example, concerning most goods, the SUB are balanced on changes in inventories (cf. Chapter 3), which implies that the additivity of the data has been verified. This is not, of course, restricted to the quarterly accounts: the annual accountants also check the balance of the SUB which they calculate in volumes at constant average prices from the previous year (not chained). But, unlike the annual accountants who work on whole years at a time, the quarterly accountants work on time series. The additivity of these series thus needs to be verified. However:

- series with chain-linked volumes based on last year’s prices are non-additive;
- series based on last year’s constant average prices (not chained) may be additive, but the sudden change observed in the first quarter of each year makes them unsuitable for use in the important econometric operation of calibration and fitting.

In theory, it would have been possible to estimate the calibrations directly using chain-linked series, then move on to volumes at the previous year’s prices (non-chained) in order to balance the input-output tables, then subsequently chain-link the resulting data. But this method would have demanded a considerable amount of programming. It thus seemed to make sense to continue using constant base-year prices and, having calculated volumes based on these prices, to make the transition to volumes based on last-year’s prices and then perform chain-linking. Initially the decision to continue using constant base-year prices will not have any noticeable effect for users, since ultimately the quarterly series published in chained volumes are absolutely consistent with the series featuring in the annual accounts.

Technically, the process is divided into three steps.

- In the first step, annual volumes at constant prices are reconstructed from the chained annual accounts (‘unchaining’). In principle, constructing constant price accounts by chain-linking volume or price indices requires to add up chained volumes with the greatest degree of precision possible. However, for reasons of simplicity of implementation, the quarterly accounts use chained volumes at a relatively aggregated level – the calibration figures, generally calculated for 48 outputs – to reconstruct volumes at constant prices using the data given in the annual accounts.
- In the second step, calibration and fitting (described previously in this Chapter) and input-output evaluation (described in Chapter 3) are performed in order to arrive at quarterly accounts expressed in volumes at constant prices derived from the base year.
- In the third step, the quarterly accounts at constant base-year prices are transformed into accounts expressed in volumes at constant average prices from the previous year. The data are then chain-linked at all levels in order to obtain the accounts which will ultimately be published.

This schematic description nonetheless runs up against a very particular problem: during the first step, to the extent that the chained volumes do not respect the accounting balance and the quarterly accounts are not based on the most detailed information but generally use 48-product aggregates, the accounts (uses-supply balance, the accounts for each branch of activity, etc.) may not be balanced on the scale at which the quarterly accounts operate. Thus for a given branch of activity, the ‘spontaneous’ volume of added value at constant prices does not correspond precisely to the difference between the volume of output at constant prices and the intermediate consumption of that branch. In the quarterly accounts calculated at constant prices, the ‘unchaining’ of the annual accounts thus involves a number of accounting items which are designed to ensure that a structural balance is achieved. This means that there will be a certain degree of deviation in the chained values of these items. To simplify: in order to balance the goods and services accounts, the deviation from the chain-linked series is ascribed to changes in inventories for goods, and output for services. To balance the intermediate input table, the deviation in the chained values is counted with the margins (total intermediate consumption by branch and total intermediate consumption by product), whereas for output account the balancing operation concerns value added, and adjusting business services allows to balance the figures out to reach a net transfer balance of zero.

In order to ensure that the published quarterly accounts are correctly calibrated with the annual accounts at the end of the third step in the process (i.e. the moment of transition from figures calculated at last year's prices), it is important to perform the reverse calculation and subtract the deviation values from the balanced items obtained by balancing the input-output tables at constant prices from the base years. As such these balanced items will be correctly calibrated with their chained-volume annual equivalents.

Observing this sequence allows to ensure that the quarterly accounts are consistent with the published national accounts at all levels: all of which means that, whatever the output or operation considered, the sum of the volumes (raw or seasonally-adjusted) for all four quarters in a given year will be equal to the annual volume published in the national accounts for that year.

Certain aggregates are generally considered to be unsuitable for chain-linking, and thus the operation is not performed. This is particularly true of the balance of trade (exports less imports), as well as changes in inventories. These two examples are instructive, as both are subject to substantial fluctuations which require frequent changes of sign.

Moreover, establishing data series for changes in inventory volumes presents at least two very specific problems:

- major fluctuations, with changes of sign and, occasionally, values which are close to zero;
- the difficulty of keeping track of price indices for each source of output, due to the fact that changes in inventories represent an adjustment item, for both value and volume.

The combination of these two problems can have drastic effects on attempts to create chain-linked series.

Another alternative is to apply an original method which can be called 'additive chain-linking with price index supervision'. This is the method proposed by Berthier [4], and used to perform chain-linking in the annual accounts (M. Braibant and C. Pilarski [6]).

2.4.3. Calculating contributions in the chained-volume accounts

The fact that the chained volume accounts are not additive makes it more difficult to calculate contributions, which will be familiar to users of the quarterly accounts. *Appendix 9* explains these difficulties, and some of the measures taken to arrive at a satisfactory definition of contributions.

2.4.4. Drawing up accounts in volumes, values and prices on an elementary level

When compiling the quarterly accounts figures in terms of volumes, values and prices, these figures are generally obtained using a volume indicator and its corresponding price indicator. Sometimes information is not available regarding volumes, or this information is not of the same quality as the information regarding prices. In such cases the quarterly accounts are compiled using a value indicator and its corresponding price indicator. It is less common to see a volume indicator and a value indicator used in conjunction on the same accounting item.

Account data are compiled using a number of standard methods.

- When dealing with a volume indicator (e.g.: number of new vehicles registered):
    - Calculation of the account volume: raw volume indicator -> seasonal adjustment (SA) and working day adjustment (WDA)-> SA-WDA volume indicator -> calibration-fitting -> final SA-WDA account volume.
    - Price indicator: raw price indicator -> seasonal adjustment[13] -> seasonally-adjusted price indicator.
    - Calculation of the account value: value indicator = account in volume * price indicator -> calibration-fitting -> account value figure.
    - Account price = value / volume.

- When dealing with a value indicator (e.g.: the turnover index):
    - First work out the account price value.
    - The account volume indicator will then correspond to the account value deflated using the price indicator.

- When dealing with a value and a volume indicator:
    - The two figures (value and volume) are calculated separately using the calibration-fitting method.
    - The account price value can be deduced from the ratio of value to volume.

2.5. Retropolation in the quarterly accounts

2.5.1. General principles applied to data from 1980 onwards

The general principles on which the quarterly accounting system is based require time series to be available over a long time period. These series are available in the 2005 base for every quarter since Q1 1980. All accounts drawn up since this date, up until the last quarter for which information is available, are published and may be subject to subsequent revisions. Generally speaking, the further back we go the smaller these revisions become. More substantial revisions are made when the first results for Q1 are published in May, integrating the data from the definitive, semi-definitive and provisional accounts published for the last three years and leading to a re-estimation of the calibration ratios as well as the SA-WDA coefficients.

In order to continue exploiting the data series which stretch back to 1980, a process of retropolation is often necessary. Taking the example of an IPI with figures dating back to 1990 published in the 2005 base, each series was back-projected for the years before 1990. This back-projection process was conducted using the old indicators originally used to compile the quarterly accounts in previous base systems, necessitating special ‘change of basis’ matrices allowing us to match old and new classification systems. These change of basis matrices are created at a relatively aggregated level, for each operation included in the accounts. It is not possible to make them as precise as the change of basis matrices used by the information suppliers themselves, which are generally applied directly at the individual level. When a systematic bias is detected in the way these aggregated matrices were applied, the retropolation of the indicators is adjusted accordingly.

Indicators are not always available for past exercises. This is particularly true of those families of indicators whose scope has been refined over time. Certain series may thus be derived from smoothing in earlier years, and calibration and fitting in more recent years. A series may also represent the results of two different successive indicators, with the second superseding the first on account of its greater accuracy but not offering the same historical coverage (this is the case for wages by branch of activity, cf. Chapter 4).

[13] Price indicators are generally impervious to working day variation, and as such are only adjusted to reflect seasonal variation.

2.5.2. A specific retropolation method for the accounts from the period 1949 to 1979

Moreover, as was also the case with the 2000 base, the 2005 base has been used to perform retropolation of the quarterly accounts beyond 1980, that is to say adapting the historical data series to correspond to the new levels and classification systems.

The methodology associated with this exercise is more straightforward than that used to estimate the current quarterly accounts, in that:

- there is no need to extrapolate data which are not yet available;
- rather than using the traditional indicators such as the IPI, we use old incarnations of the quarterly accounts as indicators, particularly since many of the indicators originally used to draw up these accounts are no longer available;
- the level of work is much more aggregated than it is when compiling the current accounts: for the year in progress, the quarterly accounts are far more detailed (including over 50 items, cf. Chapter 3) than the historic publications (17 items). For the purposes of retropolation, the level of work is directly that of publication – 17 items (A17). It is not realistically possible to work at a more detailed level since the accounts used as indicators are only archived at a certain level of detail (between 15 and 20 items), and are based on classification frameworks which differ considerably from that now used in the national accounts; for these reasons, the level of detail is that of the publication.

Nonetheless, all of the general principles used to produce the quarterly accounts are respected (calibration and fitting with annual data, SA-WDA, etc.), so much so that the pre- and post- 1980 series are consistent.

F. Arnaud and R. Mahieu [3] explain the method by which the 2000 base was retropolated back past 1980 as far as 1949, with the help of retropolated annual accounts, and thanks to quarterly accounts drawn up using the 1956 and 1970 reference bases.

The 2005 base incorporates the same principles, building on the works published in the 2000 base and using different matrices allowing to make the transition from the 16-item French composite economic classification (NES) used with the 2000 base to the aggregated 17-item classification used in base 2005.

With the exception of some rate changes made to the annual data, the pre-1980 series are no longer subject to revision.

Appendix 1: The process of selecting a calibration model

When selecting a calibration model, we must choose between the six available options summarised in the table below:

Model Number	Short description	General description: Level/variation	Constant	Estimation (1)
1	$\Delta Y_t = \alpha + \beta \Delta X_t + \varepsilon_t$	Variation	Yes	OLS
2	$\Delta Y_t = \beta \Delta X_t + \varepsilon_t$	Variation	No	OLS
3	$Y_t = \alpha + \beta X_t + \varepsilon_t$	Level	Yes	OLS
4	$Y_t = \alpha + \beta X_t + u_t$	Level	Yes	QLS
5	$Y_t = \beta X_t + \varepsilon_t$	Level	No	OLS
6	$Y_t = \beta X_t + u_t$	Level	No	QLS

(1) OLS : ordinary least squares; QLS : quasi-least squares

Where : ε_t : residual (white noise)

u_t : residual of the calibration model supposed to follow an autoregressive process of order 1.

Source: quarterly national accounts

The detailed example given below illustrates the process which led to the selection of the model used to calibrate household consumption of new cars, calculated in volumes at constant prices. The indicator comes from the French Committee of Carmakers (the CCFA), and is calculated by mutliplying two indicators: the first one is new car registrations for individuals, the second one is the engine rating of vehicles for tax purposes. The following table presents the results obtained from all 6 of the models described above, used to calibrate the account figures on the basis of this indicator for the period 1980-1990.

Table: results from different models, applied to the example of household expenditure on new cars

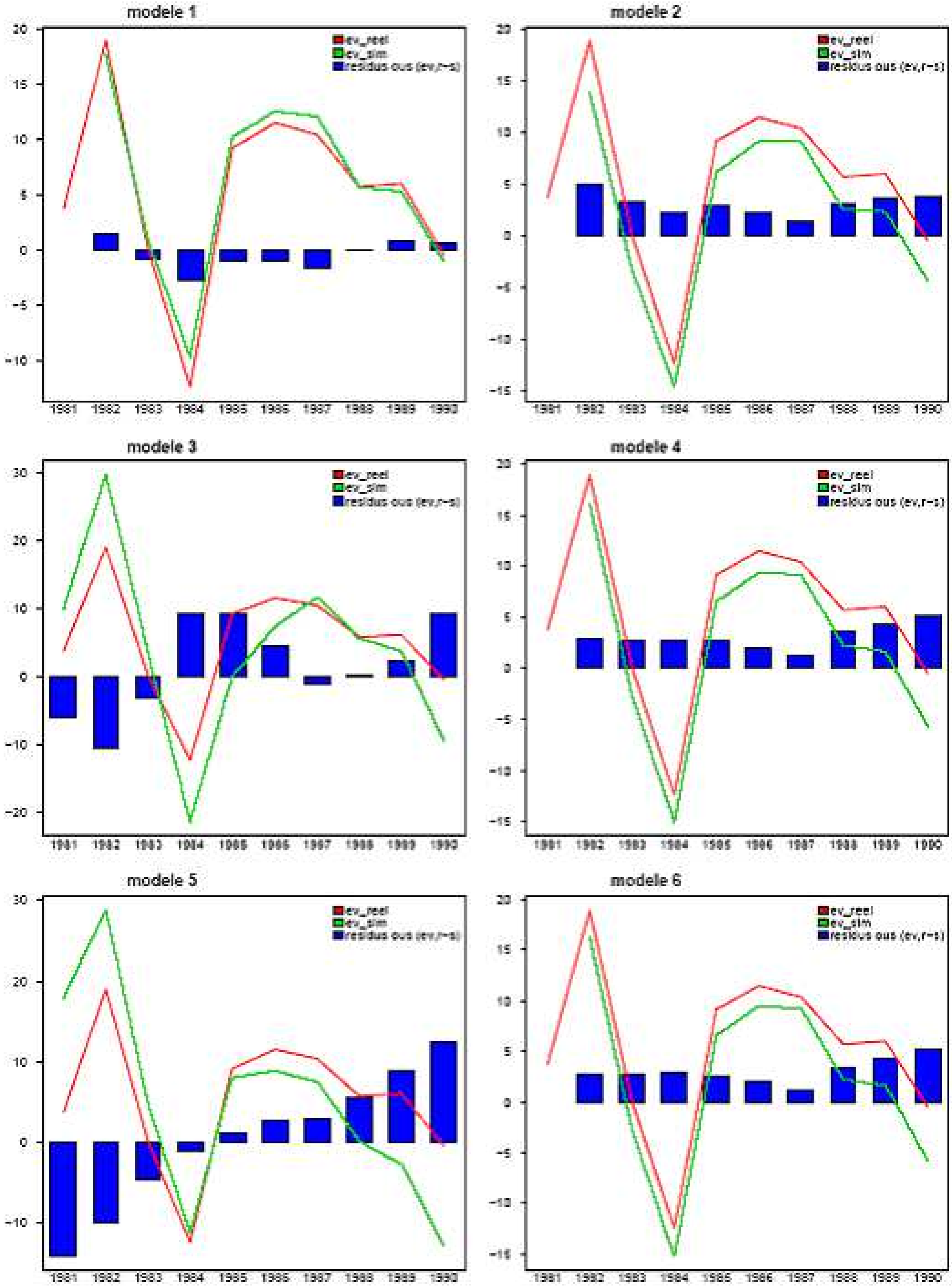
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Recursive residuals (RMSE)	0.7	3.6	8.9	5.1	12.2	5
Ljung Box test L = 1 (p.value)	0.2	0.28	0.17	0.27	0.02	0.3
Coefficients						
constant	65.92	0	-1029.61	22.52	0	0
(Student)	(5.85)	(0)	(-2.07)	(0.07)	(0)	(0)
indicator	1.87	2.02	3.44	2.14	2.19	2.16
(Student)	(11.13)	(5.62)	(5.68)	(5.91)	(33.7)	(15.81)
rho	0	0	0	0.88	0	0.88

Source: quarterly national accounts

Models 3 and 5 are level models without autocorrelation of the calibration residual, with and without a constant. The Ljung Box test (a variation of the portmanteau test which allows to assess the autocorrelation of the residuals) refutes Model 5 (low probability, the residual is not a white noise). Models 4 and 6 include autocorrelation of the calibration residual, with and without a constant; for Model 4 the Student test suggests that the constant is not statistically significant, making Model 6 the more attractive of the two options. Models 1 and 2 are differential models, with and without a constant; the Student test shows that Model 1 achieves a significant constant, and that the most suitable option is Model 1 with the constant.

Ultimately a choice needs to be made between Models 1, 3 and 6. In the table, the standard deviation of the recursive residuals shows that Model 1 has the strongest predictive capacity. Moreover, the subsequent graphs show that the prediction error is lower with Model 1, and that this prediction error is not biased (the graphs allow to compare the real evolution of the data series against the projections generated by the model; the bars represent the difference between the two, i.e. the prediction error for each year). For the period under consideration, Model 1 is thus the best option. We should also note that additional tests such as the unit root test allow to determine whether the account series and indicators are integrated or stationary, and thus allow to justify our choice of Models 1 and 2 ahead of the other available options.

Graph: Results of 'in sample' estimates for the various models, using the example of household expenditure on new cars 1980-1990



How to read it: for excel model graph, evolution of annual account (in red) is compared to estimation by the model (in green)

Source: quarterly national accounts

Appendix 2: The Portmanteau test

This test is conducted on the random values contained in the calibration equation. It involves estimating the autocorrelation coefficient of these random variables using the equation:

$$\varepsilon_t = \rho \times \varepsilon_{t-1} + \eta_t$$

Which gives us:

$$\hat{\rho} = \frac{\mathrm{cov}(\varepsilon_t, \varepsilon_{t-1})}{\mathrm{var}(\varepsilon_t)}$$

and, assuming that η_t is normal:

$$\hat{\rho} \approx \mathrm{N}\left(\rho, \frac{\sigma_\eta^2}{\sum \varepsilon_t^2}\right)$$

Where $\rho = 0$, and T represents the number of observations, we are left with:

$$\hat{\rho} \approx \mathrm{N}(0, \frac{1}{T})$$

the portmanteau test (Ljung-Box method) thus holds that:

$$\hat{\rho}^2 * T \approx \chi^2(1)$$

Appendix 3: The smoothing procedure

The procedure for 'smoothing' an annual data series X_a basically involves estimating a quarterly series $X_{a,t}$ where the squared deviation from one quarter to the next is minimal, bearing in mind that for each year a the sum of the individual quarters must be equal to the annual total X_a :

$$\min \sum_t (X_{a,t} - X_{a,t-1})^2$$

$$\text{with the constraint that: } \sum_{t=1}^{4} X_{a,t} = X_a \text{ for } a = 1...A$$

To avoid generating discontinuity at the end of the quarterly series, it is important to extrapolate the annual series over several years. Smoothing is thus performed for the whole annual series, including the extrapolated years. In the quarterly accounts smoothing is applied to the calibration residuals, and to the annual accounts for which there is no indicator. In both cases, the annual data are extrapolated for two years and then smoothed.

This programme has a closed-form solution. The notation used is as follows:

- $x = (X_{1,1},.., X_{1,4},.., X_{A,1},.., X_{A,4})'$, the vector of the quarterly series with the dimension $[4A,1]$

- $X = (X_1,.., X_A)'$, the vector of the annualised series with the dimension $[A,1]$

The programme is expressed in matrix form:

$$\begin{cases} \min\limits_{x} (Dx)'(Dx) \\ st \quad Mx = X \end{cases}$$

where D represents the matrix

$$D = \begin{pmatrix} -1 & 1 & 0 & & & & \\ 0 & -1 & 1 & 0 & & & \\ & 0 & \cdots & \cdots & & & \\ & & & \cdots & \cdots & 0 & \\ & & & 0 & -1 & 1 & 0 \\ & & & & 0 & -1 & 1 \end{pmatrix} \text{ with the dimension } [4A-1,4A]$$

and *M* represents the matrix defined by:

$$M = I_A \otimes (1 \quad 1 \quad 1 \quad 1) \text{ with the dimension } [A,4A].$$

y is a variable defined by $y = Dx$. The minimisation programme thus becomes:

$$\begin{cases} \min\limits_{x_{1,1},y} y'y \\ st \quad MT\begin{pmatrix} x_{1,1} \\ y \end{pmatrix} = X \end{cases}$$

where $T = \begin{pmatrix} 1 & 0 & & & \\ 1 & 1 & & & \\ 1 & 1 & 1 & 0 & 0 \\ & & & 1 & 0 \\ 1 & & & 1 & 1 \end{pmatrix}$ with the dimension $[4A,4A]$

The Lagrangian associated with this programme is:

$$\begin{aligned} L &= y'y + 2\lambda'\left[MT\begin{pmatrix} x_{1,1} \\ y \end{pmatrix} - X \right] \\ &= y'y + 2\lambda'\left[m_1 x_{1,1} + \tilde{m}y - X \right] \end{aligned}$$

where

$m_1 = MT\begin{pmatrix} 1 \\ 0 \\ \dots \\ 0 \end{pmatrix}$ and $\tilde{m} = MT\begin{pmatrix} 0 \\ I_{4A-1} \end{pmatrix}$

By differentiating with respect to $x_{1,1}, y\ et\ \lambda$, we calculate that

$$(1)\ \lambda' m_1 = 0$$
$$(2)\ y = \tilde{m}'\lambda$$
$$(3)\ m_1 x_{1,1} + \tilde{m}y = X$$

By combining (2) and (3), we can deduce the value of λ as a function of $x_{1,1}$:

$$(3')\ \lambda = \left(\tilde{m}\tilde{m}'\right)^{-1}\left(X - m_1 x_{1,1}\right)$$

Starting with (1) and (3'), we can calculate $x_{1,1}$:

$$x_{1,1} = m_1'\left(\tilde{m}\tilde{m}'\right)^{-1} X \ / \ m_1'\left(\tilde{m}\tilde{m}'\right)^{-1} m_1$$

Using (3') we can deduce λ and using (2) we can calculate the value of y. We can then deduce x because

$$x = T\begin{pmatrix} x_{1,1} \\ y \end{pmatrix}$$

Appendix 4: The rate smoothing procedure

Sometimes, in the absence of sufficient information, we may decide to smooth not just an account total but also a related rate, for example an operating margin, a tax rate or a price. We then assume that an account T is economically equivalent to a rate tx multiplied by an account C. The rate smoothing procedure basically involves smoothing the annual rate and defining the quarterly account $T_{a,t}$ using the equation:

$$T_{a,t} = tx_{a,t} \times C_{a,t} \,.$$

The quarterly rate is the result of the following minimisation programme:

$$\min \sum_{t} (tx_{a,t} - tx_{a,t-1})^{2}$$

With the constraint: $\sum_{t=1}^{4} tx_{a,t} \times C_{a,t} = tx_{a} \times C_{a} = T_{a}$ for $a = 1 ... A$

This constraint may also be expressed:

$$\sum_{t=1}^{4} tx_{a,t} \times \left(\frac{C_{a,t}}{C_{a}}\right) = tx_{a}$$

At a more general level the programme can be expressed thus:

$$\begin{cases} \min \sum (X_{a,t} - X_{a,t-1})^{2} \\ st \\ \sum_{t} X_{a,t} p_{a,t} = X_{a} \text{, for } a = 1...A \end{cases}$$

This equation can be solved in exactly the same way as the usual smoothing programme, by replacing the matrix M with

$$M = \begin{pmatrix} p_{1,1} & p_{1,2} & p_{1,3} & p_{1,4} & & & & \\ 0 & 0 & 0 & 0 & & & & \\ & & & & & & & \\ & & & & 0 & 0 & 0 & 0 \\ & & & & p_{A,1} & p_{A,2} & p_{A,3} & p_{A,4} \end{pmatrix}$$

Appendix 5: The method used to assess the impact of variation in the number of working days

We can postulate the following relationship between a raw variable and its WDA equivalent, with ‘a’ representing the year and ‘m’ the month:

$$Y_{a,m} = \sum_{i=1}^{7} \alpha_i N_{a,m}^{i} + \beta F_{a,m} + \gamma \delta_{m,2} \delta_{a,biss} + Y_{a,m}^{WDA} \qquad (1)$$

where:

$Y_{a,m}$ represents the raw variable

$Y_{a,m}^{WDA}$ represents the variable adjusted for working day variation

$N_{a,m}^{i}$ represents the number of working days of type i (i=1,…,6) in the month under consideration,

and $N_{a,m}^{7}$ represents the number of Sundays in this same month (whether or not they are public holidays)

$F_{a,m}$ represents the number of public holidays (excluding Sundays) in this month

$\delta_{m,2}\delta_{a,biss}$ is a dummy which takes on a value of 1 in the month of February, but only for leap years.

The WDA data series $Y_{a,m}^{WDA}$ is thus the residual of the estimated econometric equation.

Calendar-related variables are first centered at the seasonal mean so as to exclude all seasonal influence (this does not fundamentally change the nature of the model).

Furthermore, in order to handle series which do not include a seasonal component and which are stationary (as far as possible), the model is estimated using the year-on-year method:

$$\Delta_{12} Y_{a,m} = \sum_{i=1}^{7} \alpha_i \Delta_{12} N_{a,m}^{i} + \beta \Delta_{12} F_{a,m} + \gamma \delta_{m,2} I_a + \Delta_{12} Y_{a,m}^{WDA} \qquad (2)$$

where: $I_a = 1$ if *a* is a multiple of 4

$I_a = -1$ if *a-1* is a multiple of 4

$I_a = 0$ in all other cases.

However, the total number of working days, Sundays and public holidays not falling on a Sunday is equal to the number of days in a month. For example, for the month of January (m=1):

$$\forall a , \sum_{i=1}^{7} N_{a,1}^{i} + F_{a,1} = 31$$

February is different, for the obvious reason that the number of days oscillates between 28 and 29. We can express this as:

$$\sum_{i=1}^{7} \Delta_{12} N_{a,m}^{i} + \Delta_{12} F_{a,m} = \delta_{m,2} I_a \qquad \text{(J)}$$

The variable $\delta_{m,2} I_a$ should thus be seen as a corrective element related to the existence of leap years.

We are then presented with a collinearity problem, as a linear relationship can be observed between the different explanatory variables. We can solve this problem by integrating the collinear relationship directly into the regression, giving us:

$$\Delta_{12} Y_{a,m} = \sum_{i=1}^{7} \varphi_i \Delta_{12} N_{a,m}^{i} + \theta\delta_{m,2} I_a + \Delta_{12} Y_{a,m}^{WDA} \qquad (3)$$

where $\varphi_i = \alpha_i - \beta$ and $\theta = \gamma + \beta$

The impact of the number of public holidays is thus non-identifiable.

The coefficients φ_i must then be interpreted with reference to these public holidays. As such the coefficient for working Mondays does not correspond to the gross effect of a working Monday, but to the effect of this day relative to the effect of a public holiday. For example, household expenditure on “accomodation and food services” is greater on public holidays than it is on working days. The coefficients attached to the number of working days are thus negative.

- **Constructing the N_t^i variables**

In order to ensure that the estimate preserves those effects ascribable uniquely to working day variation, working days variables are seasonally adjusted by a process of standard deviation, using a Buys-Ballot table (*Appendix 6*).

- **Selecting the parameters of the model**

There are two ways of assessing the impact of working day variation: an additive method, and a multiplication method. The latter involves estimating the impact of working days on the logarithm of the indicator. This supposes that the impact is proportional to the level of the indicator; the larger the indicator, the greater the calendar effect. The choice of model is made jointly for both adjustment operations (WDA and SA).

To choose between the additive and multiplication methods, an initial graphical analysis of the series is conducted in order to determine whether or not the seasonal variation increases in line with the underlying trend. To put it another way, the objective is to determine whether this seasonal variation is essentially multiplicative. To confirm the results of this analysis, a simple econometric test is carried out, constructing a regression of the annual amplitudes against their averages. A significant coefficient will point out that the multiplication method is the best option. Thereafter, a stability test on the working day coefficients allows to identify any major instabilities which may result from a poor choice of method. More complex tests do exist (for example the nonlinear Box-Cox test) but are not used here, not least due to the calculation and analysis time required to apply them correctly.

For the rest of *Appendix 5*, the explanation of the working day adjustment method continues, based on the assumption that an additive method has been chosen. To make the transition to a multiplication method the indicator simply needs to be replaced by its logarithm.

- **Step one: degree of differencing**

The model is estimated using the year-on-year method in order to obtain stationary data. Nonetheless, the year-on-year evolution of the indicator may not be stationary. A Dickey-Fuller test is performed on the residual (3). If the nonstationary hypothesis is refuted, the first difference of the equation (3) is calculated.

- **Step two: detecting the influence of working day variation**

This involves conducting a Fisher test on the parameters of the model (3) (differenced where relevant). If the test finds out that the coefficients are not jointly significant, no working day adjustment is required for this indicator.

- **Step three: detecting the existence of a leap year effect**

This step involves carrying out a Student test on the θ parameter of the leap year dummy. If the test finds out that a coefficient is not significant, or conversely has a value which is out of proportion with that observed for other days, no leap year adjustment is performed on this indicator.

- **Step four: testing for specific effects for July and August**

The months of July and August can undermine attempts to estimate the effects of working day variation: during these months many people take their holidays, which can have the effect of attenuating the influence of working days. To test the effects of calendar variations in the months of July and August, the effects are first assessed for the summer months only. A Fisher test then allows us to determine whether or not the coefficients are significant. If they are not, then the coefficients for the months of July and August must be null, i.e. no working day effects are subtracted from the indicator for the summer months; the working day coefficients are then estimated for all other months. If the test finds that calendar variations do have a significant effect in the summer months, then the coefficients are assessed in the same way for all months of the year, with no specificity for summer months.

- **Step five: grouping tests**

These five tests allow to draw up groups. The most practical approach is to use very limiting specifications, which contribute to better understanding the coefficients used and to improving the precision of the estimated effects. Each test corresponds to a Fisher test on specific coefficients. These tests are performed using a model in which the residuals follow an order 1 autoregressive model.

The grouping possibilities tested are:

- the Sunday coefficient must be zero (this is often the case for consumption indicators);
- the coefficients corresponding to the days of the week must be equal, and the Sunday coefficient must be zero (another common scenario for consumption indicators);
- the Saturday and Sunday parameters must both be zero (this is more applicable to output indicators);
- the coefficients corresponding to the days of the week must be equal, and the Saturday and Sunday coefficient must both be zero (this is also applicable to output indicators);
- the coefficients corresponding to the days of the week must be equal.

- **Step six: compensation and forward planning**

Once the grouping option has been chosen, the next step is to determine whether extending the explanatory variables to the months immediately before and after the month in question has a significant impact. It is in fact possible that companies will compensate for the effect of a public holiday by increasing their output over the subsequent weeks, or conversely they may anticipate by increasing production in the weeks leading up to the holiday. It is also possible that household expenditure on clothing may be spread over several weeks when, for example, a public holiday falling on a Saturday causes a significant drop in consumption for that day. In both cases, the equation (3) is estimated using not only the working day coefficients for the month at hand, but also those for the months immediately before and after. A Fisher test on the lead and lag variables, and both together, is then carried out.

- **Step seven: stability tests**

It is important to test the stability of the estimated coefficients. Changes in behaviour can possibly modify the impact of public holidays on the French economy. For example, the fact that shops are now increasingly open on Sunday and public holidays will probably modify the effect that these days have on household consumption.

Chow tests are performed for each year. If significant fluctuations appear, the working day coefficients are calculated using sliding scales covering several years. Determining the number of years to be used in these scales is a delicate operation: they must be sufficiently small to adapt to fluctuations, yet broad enough to allow to estimate the coefficients to an acceptable degree of accuracy.

Appendix 6: Seasonal adjustment using standard deviation (Buys-Ballot)

The Buys-Ballot method is a seasonal adjustment technique based on standard deviation. The method consists of estimating the seasonal effect of a quarter, represented by the mean value of the data series for this quarter, on all years for which observations are available, reduced by the overall mean.

The method involves conducting a regression of the series to be adjusted $I_{a,t}$ against the quarterly indicators:[14] $\delta^1_{a,t}$, $\delta^2_{a,t}$, $\delta^3_{a,t}$, $\delta^4_{a,t}$, where, for example, $\delta^1_{a,t} = 1$ if t is a first quarter ($t = 1$), 0 otherwise:

$$I_{a,t} = \alpha_1 \times \delta^1_{a,t} + \alpha_2 \times \delta^2_{a,t} + \alpha_3 \times \delta^3_{a,t} + \alpha_4 \times \delta^4_{a,t} + \varepsilon_{a,t}$$

The model is estimated without a constant, because the sum of the indicators will be 1 regardless of the date.

The estimated coefficients are in fact equal to the mean values of the series for each quarter: $\hat{\alpha}_1 = \bar{I}_{\{T1\}}$, ..., $\hat{\alpha}_4 = \bar{I}_{\{T4\}}$

where $\bar{I}_{\{T1\}} = \frac{1}{A}\sum_a I_{a,1}$ and A is the number of years.

The sum of the coefficients is proportional to $\bar{I}$, the overall average of the series: $\sum_{i=1}^{4} \hat{\alpha}_i = 4\bar{I}$.

The seasonal coefficients are ultimately defined as the deviation between the mean for each quarter and the overall mean:

$a_1 = \hat{\alpha}_1 - \bar{I} = \bar{I}_{\{T1\}} - \bar{I}$, ..., $a_4 = \hat{\alpha}_4 - \bar{I} = \bar{I}_{\{T4\}} - \bar{I}$.

The seasonally-adjusted series is thus:

$$I^{cvs}_{a,t} = I_{a,t} - a_1 \times \delta^1_{a,t} - a_2 \times \delta^2_{a,t} - a_3 \times \delta^3_{a,t} - a_4 \times \delta^4_{a,t}$$

Seasonal adjustment is neutral over the whole year, since the coefficients confirm that: $\sum_{i=1}^{4} a_i = 0$.

[14] For the purposes of this example we assume that the series $I_{a,t}$ is stationary. If $I_{a,t}$ follows a linear progression, the regression model used will be: $I_{a,t} = \lambda + \beta \times (4 \times a + t) + \alpha_1 \times \delta^1_{a,t} + \alpha_2 \times \delta^2_{a,t} + \alpha_3 \times \delta^3_{a,t} + \alpha_4 \times \delta^4_{a,t} + \varepsilon_{a,t}$. This model is not immediately identifiable. A constraint therefore needs to be added to the coefficients (e.g. $\sum_{i=1}^{4} \alpha_i = 0$)

Appendix 7: The X12-ARIMA seasonal adjustment method

The method used by the X12 programme consists of extracting from a data series I_t (already adjusted for working day variation) a component corresponding to the trend-cycle TC_t, a seasonal component S_t and an irregular component R_t. The seasonal adjustment consists of estimating the impact of the seasonal component in order to correct the initial data series of this component.

It is important to note that, in conformity with the concepts used in the quarterly accounts, the irregular component R_t must feature in the seasonally-adjusted data series, and thus in the SA-WDA series as well. For example, the effects of a strike on a given branch of activity must be clearly visible in the SA-WDA output figures for this branch.

The X12 method is based on a number of moving averages, which are applied to the series. The seasonal adjustment of a quarterly data series I_t involves three key steps:

Step 1: initial estimation of the seasonal and irregular components

Using a symmetrical moving average $M(.)$, it is possible to separate the trend cycle component (TC_t) and the seasonal-irregular component (S_t+R_t):

$$M(I_t) = \frac{1}{8}(I_{t-2} + 2I_{t-1} + 2I_t + 2I_{t+1} + I_{t+2})$$

$I_t - M(I_t) = \left(\hat{S}_t + \hat{R}_t\right)$ thus gives an estimation of the combined value of the seasonal and irregular components (S_t+R_t).

Step 2: first estimation of the seasonal component

In order to remove the irregular component from $\left(\hat{S}_t + \hat{R}_t\right)$, for each quarter we apply a moving average M' defined as:

$$M'(I_t) = \frac{1}{9}(I_{t-8} + 2I_{t-4} + 3I_t + 2I_{t+4} + I_{t+8}) \quad \text{(5-year average)}$$

Thus $M'(I_t - M(I_t)) = \hat{S}_t$ gives an estimate of the seasonal component.

Step 3: standardising the seasonal component only

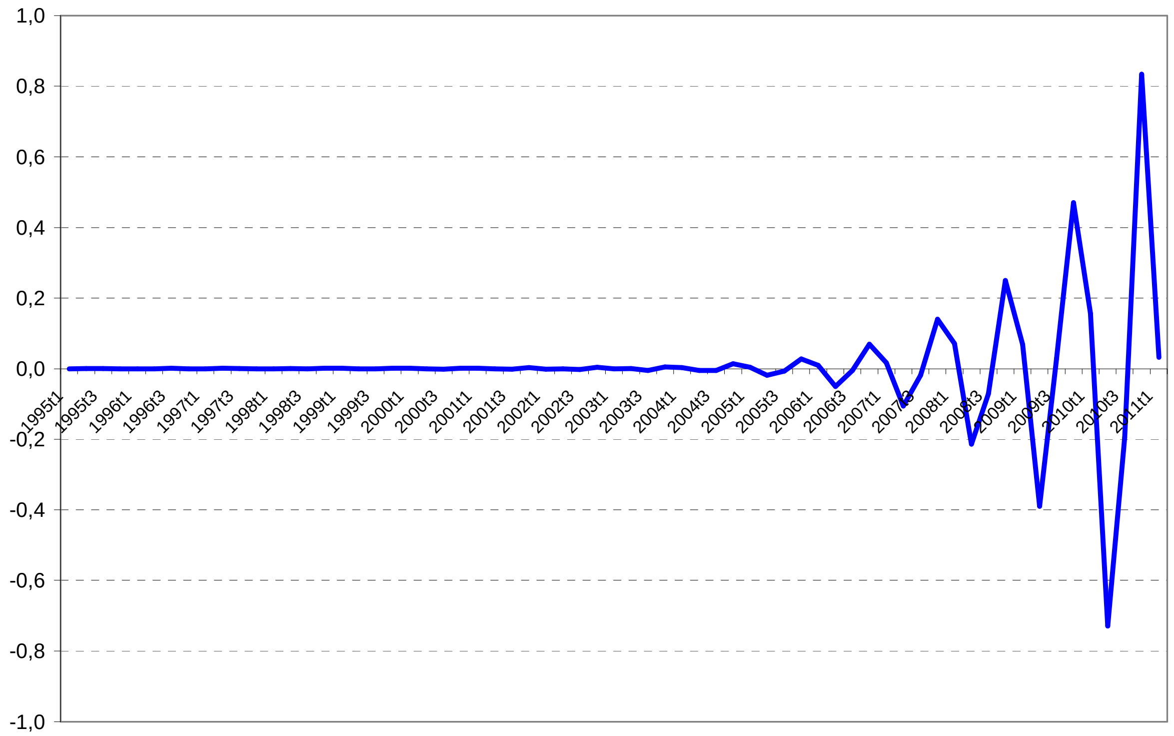
However, this estimate of the seasonal component does not satisfy the constraint of a zero sum for each year. We thus impose another constraint: if t is one of the four quarters of the year a and $\overline{S_a}$ is the sum of $\hat{S}_t$ for the year a, we can define $S_t^{'}$ as the standardised seasonal component thus $S'_t = \hat{S}_t - \frac{1}{4}\overline{S_a}$.

We thus obtain an initial estimate of the seasonally-adjusted series: $I_t - \hat{S}'_t = \hat{I}_t^{cvs}$.

In order to refine this initial estimate, the software repeats these three steps using other moving averages (including Henderson moving averages), applying the same principles. We thus arrive at a second estimate of the seasonal coefficients, which we once more transform to ensure that they satisfy the zero sum requirement. By removing this coefficient from the initial series, we arrive at the final, seasonally-adjusted values.

In order to apply these moving average filters at the start and the end of the period, and thus to avoid any deformations at the 'fringes' of the series, we must perform both retropolation and extrapolation. The method used by X12-ARIMA involves choosing an ARIMA model which accurately represents the series, then using this as a predictive model to extrapolate the series. This leads to revisions in the SA-WDA series: when a new point becomes available (or a recent value is revised), the ARIMA extrapolation will also be modified; as a result of these two factors, the seasonal effect is estimated differently. The revision itself thus takes on a seasonal nature, and its effects decrease the further back we go (see *Graph* below). Moreover, in an improvement on the previous version X-11, the X12-ARIMA programme has a special module for detecting and correcting 'outliers'. This allows to remove anomalous values (which may be due to specific short-term factors or reflect long-term changes of level) so as not to upset our calculation of the 'trend-cycle' and seasonal components.

Graph: Revising the % trend of 'food' imports, between the detailed results for Q2 2011 and those for Q1 2011



Source: quarterly national accounts

Appendix 8: From volumes at constant prices to chained volumes at last year's prices (and vice versa)

The value at a basic level for a given year n is equal to the product of a price and the quantity of this product:

$$val_n = p_n \times q_n$$

where p_n is the price and q_n the quantity.

The volume at constant prices is defined as the product of a price p_0 in the base year, and of the quantity, so that for the base year the volume is equal to the value:

$$vol_n = p_0 \times q_n$$

The price index is thus defined as the ratio between value and volume:

$$IndP_n = \frac{val_n}{vol_n} = \frac{p_n}{p_0}$$

It therefore represents the difference between the price for the year at hand and the reference price from the base year.

If we consider two elementary output values i and j, the aggregation of these two outputs is simple: the volume (and the value) of the aggregate will be the sum of the volumes (and the values) of each product.

$$vol_n(i+j) = p_0(i) \times q_n(i) + p_0(j) \times q_n(j)$$

and $val_n(i+j) = p_n(i) \times q_n(i) + p_n(j) \times q_n(j)$.

As such, the volume indicator, that is to say the evolution in volume between the base year and the year under consideration, can be expressed as:

$$IndVol_n(i+j) = \frac{vol_n(i+j)}{vol_0(i+j)}$$

or

$$IndVol_n(i+j) = \frac{q_n(i)}{q_0(i)} \times \frac{vol_0(i)}{vol_0(i+j)} + \frac{q_n(j)}{q_0(j)} \times \frac{vol_0(j)}{vol_0(i+j)}$$

As the volumes are equal to the values for the base year, the volume index involves weighting the elementary volume indices against the value structure of the base year (making it a Laspeyres index).

Generally speaking, we can express the volume index of an aggregate of elementary levels i as:

$$IndVol_n(\sum i) = \frac{\sum_i vol_n(i)}{\sum_i vol_o(i)} = \sum_i \frac{q_n(i)}{q_0(i)} \times \left(\frac{p_0(i) \times q_o(i)}{\sum_j p_0(j) \times q_o(j)} \right)$$

Volumes are generally measured either in last year's prices or at the prices of the base year. The further we get from the base year, the greater the difference will be between last year's prices and those of the base year.

However, volumes expressed at last year's prices cannot be used directly in a time series. If we compare the volume level of Year 1 and the volume level of Year 2, expressed in last year's prices, we are effectively comparing:

$$vol_1(\sum i) = \sum_i p_0(i) \times q_1(i) \text{ et } vol_2(\sum i) = \sum_i p_1(i) \times q_2(i)$$

so the difference between these two volumes reflects both a price change (between Year 0 and Year 1) and a volume change (between Year 1 and Year 2).

On the other hand the volume index for Year 2, expressed at last year's prices, does correspond to an evolution in volumes between Year 1 and Year 2:

$$IndVol_2(\sum i) = \frac{\sum_i p_1(i) \times q_2(i)}{\sum_i p_1(i) \times q_1(i)}$$

The purpose of chained indices is thus to retain the volume index estimated at last year's prices as the indicator of the change in volume, while chain-linking the available indices starting from a given reference year.

Thus if the reference year is Year 0, the chained volume for Year 2 is equal to:

$$vch_2(\sum i) = vol_0 \times IndVol_1 \times IndVol_2 = \left(\sum_i p_0(i) \times q_0(i)\right) * \frac{\sum_i p_0(i) \times q_1(i)}{\sum_i p_0(i) \times q_0(i)} * \frac{\sum_i p_1(i) \times q_2(i)}{\sum_i p_1(i) \times q_1(i)}$$

which can be simplified as:

$$vch_2(\sum i) = \frac{\sum_i p_0(i) \times q_1(i)}{\sum_i p_1(i) \times q_1(i)} * \sum_i p_1(i) \times q_2(i)$$

When we chain-link volumes at an elementary level, we obtain the volume at a constant price, that of base Year 0:

$$vch_2(i) = \frac{p_0(i) \times q_1(i)}{p_1(i) \times q_1(i)} \times p_1(i) \times q_2(i) = p_0(i) \times q_2(i)$$

By developing this formula, or applying it to a simple numerical example, we can observe that:

$$vch_2(i+j) \neq vch_2(i) + vch_2(j)$$

To put it simply, the additivity is lost. To obtain chained volumes we thus need to chain the indices at all levels of aggregation, and for all operations.

Conversely, based on the chained prices for the previous year we can reconstruct volumes at constant prices by basing our calculations on chained volumes at the most detailed level possible.

Appendix 9: Calculating contributions in chained volumes

In the context of the volumes calculated at the constant prices of the base year, a single and relatively intuitive formula allows to calculate the effect of a given variable Y on the change in a second variable X, at both annual and quarterly levels, represented in the following '*Contrib'* function:

$$Contrib\left(Y_a^{vol\ price\ ct}, X_a^{vol\ price\ ct}\right) = ev\left(Y_a^{vol\ price\ ct}\right)\frac{Y_{a-1}^{vol\ price\ ct}}{X_{a-1}^{vol\ price\ ct}}$$

$$Contrib\left(Y_t^{vol\ price\ ct}, X_t^{vol\ price\ ct}\right) = ev\left(Y_t^{vol\ price\ ct}\right)\frac{Y_{t-1}^{vol\ price\ ct}}{X_{t-1}^{vol\ price\ ct}}$$

where ev(Y) represents the growth rate of the variable Y in relation to the preceding period.

The contribution of the variable Y to the evolution of the variable X thus corresponds to the growth rate of the variable Y weighted to reflect the relative weight of Y in X in terms of volumes recorded in the previous period;

In the annual data, this formula remains valid for chained volumes requiring only a slight modification. The contribution of the variable Y to the evolution of the variable X then corresponds to the growth rate of the variable Y weighted to reflect the relative weight of Y in X in terms of **values** recorded in the previous period.

$$\overline{Contrib}_a\left(Y^{vol\ ch}, X^{vol\ ch}\right) = ev\left(Y_a^{vol\ ch}\right)\frac{Y_{a-1}^{value}}{X_{a-1}^{value}}$$

Calculating quarterly contributions, however, is a more complex undertaking which requires to make certain choices. First and foremost, simply transposing the annual formula given above has been found to be unsatisfactory (although it does preserve the additivity of the contributions made by different components of an aggregate), as this would cause a sudden jump in the first quarter (since in the first quarter the 'previous year' is no longer the same as it was for the preceding quarter).

The French quarterly accounts thus base their contribution calculations on a formula derived from the annual formula, but adapted so as to prevent sudden jumps in each first quarter, while also preserving the additivity of the contributions made by different components of an aggregate. The generic formula, which thus incorporates a corrective element specific to first quarter results, can be written as follows:

$$\overline{Contrib}_{a,t}\left(Y^{vol\ ch}, X^{vol\ ch}\right) = \begin{cases} contrib\left(Y_{a,t}^{vol\ ch}, X_{a,t}^{vol\ ch}\right) \\ + contrib\left(Y_{a,t}^{vol\ ch}, X_{a,t}^{vol\ ch}\right)\left(\dfrac{Y_{a-1}^{price}}{X_{a-1}^{price}} - 1\right) \\ + 1_{t=1} \times \left(\dfrac{Y_{a-1,4}^{vol\ ch}}{X_{a-1,4}^{vol\ ch}} - \dfrac{Y_{a-1}^{vol\ ch}}{X_{a-1}^{vol\ ch}}\right) \times \left(\dfrac{Y_{a-1}^{price}}{X_{a-1}^{price}} - \dfrac{Y_{a-2}^{price}}{X_{a-2}^{price}}\right) \end{cases}$$

The first term corresponds to the 'traditional' calculation of contributions (using the *Contrib* function). The second term introduces a corrective element connected to the price disconnect between component Y and aggregate X. The third term applies only when calculating the contributions made in the 1st quarter of each year, correcting any sudden jolts resulting from the change of reference prices (from year Y-2 to year Y-1).

At first sight this formula can seem complex. But this complexity needs to be put into perspective: for one thing, the formula is relatively easy to program.[15] Furthermore, the fact that the formula respects the additivity of the component variables means that we can also use it to calculate the contributions of those variables which, as they are sometimes negative and sometimes positive (such as changes in inventories or the balance of trade), are not available as chained volumes. The contribution of the balance of trade to GDP growth in chained volumes is thus defined as the difference between the respective contributions of exports and imports in chained volumes, despite the fact that the trade balance variable itself is not even available in chained volumes.

[15] The ‘Quarterly Accounts/ methodology’ section of the INSEE website includes a detailed explanation of the method used to calculate contributions in chained volumes, along with an Excel spreadsheet in which the contribution calculation formula has been pre-programmed using sample data.